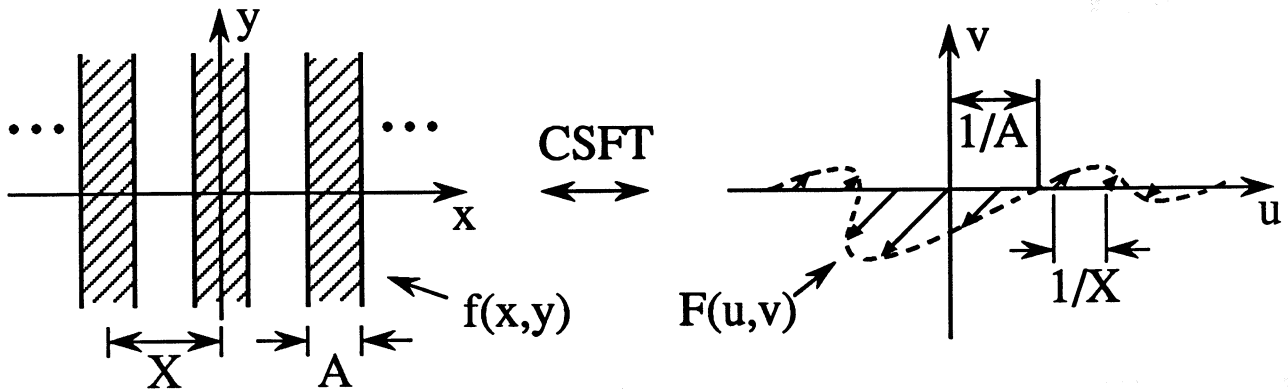
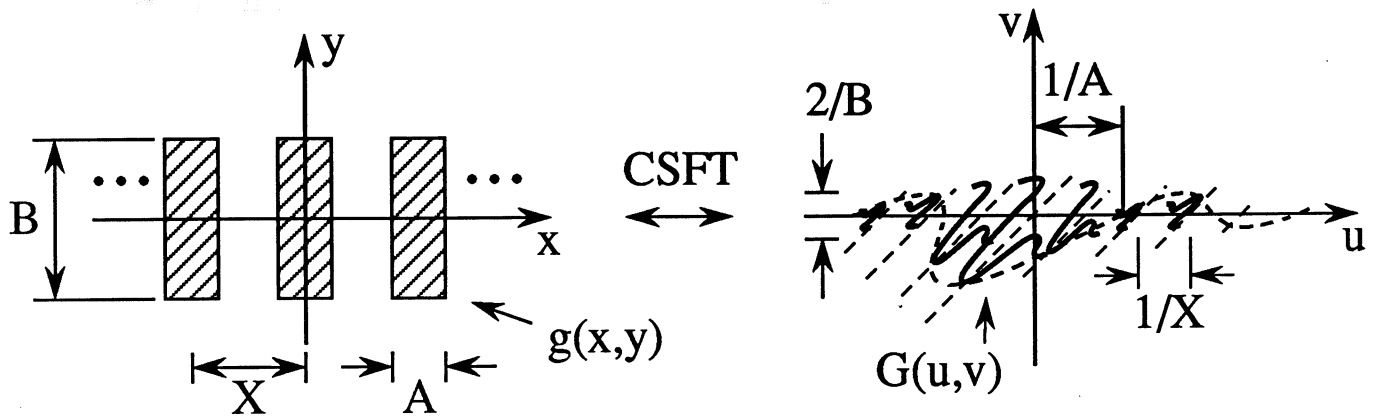


## 2.1.4 PERIODIC STRUCTURES



$$f(x,y) = \text{rep}_X[\text{rect}(x/A)] \cdot 1$$

$$F(u,v) = \frac{1}{X} \text{comb} \frac{1}{X} [A \text{sinc}(Au)] \delta(v)$$



$$g(x,y) = \left[ 1 \cdot \text{rect}(y/B) \right] f(x,y)$$

$$G(u,v) = \delta(u) B \text{sinc}(Bv) ** F(u,v)$$

$$F(u, v) = \frac{1}{X} \text{comb} \frac{1}{X} [A \text{sinc}(Au)] \delta(v)$$

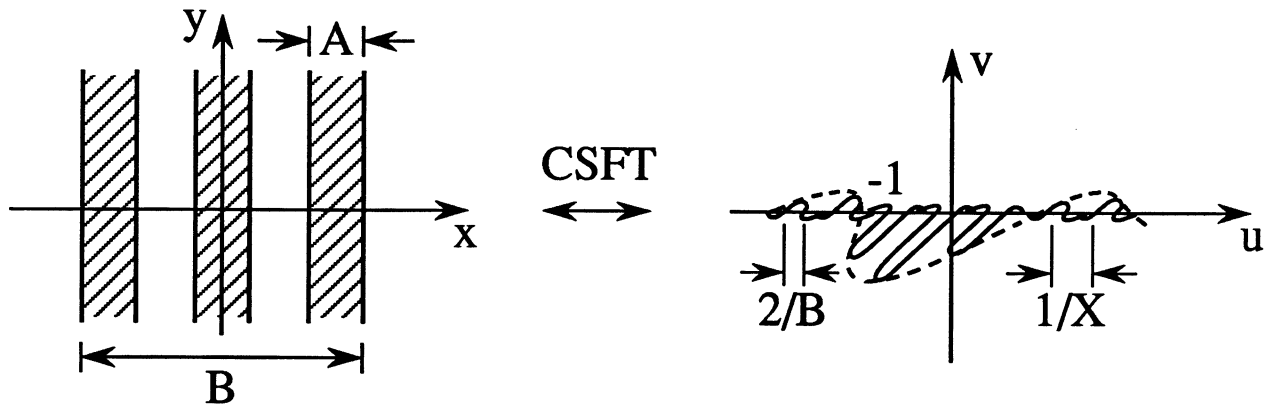
$$= \frac{A}{X} \sum_k \text{sinc} \left( \frac{A}{X} k \right) \delta \left( u - \frac{k}{X}, v \right)$$

$$G(u, v) = \int \int \delta(u - \mu) B \text{sinc} [B(v - \nu)] F(\mu, \nu) d\mu d\nu$$

$$= \frac{A}{X} \sum_k \text{sinc} \left( \frac{A}{X} k \right) \int B \text{sinc} [B(v - \nu)]$$

$$\times \int \delta(u - \mu) \delta(\mu - \frac{k}{X}, \nu) d\mu d\nu$$

$$G(u, v) = \frac{A}{X} \sum_k \text{sinc} \left( \frac{A}{X} k \right) B \text{sinc} (Bv) \delta \left( u - \frac{k}{X} \right)$$



$$g(x, y) = [\text{rect}(x/B) \cdot 1] f(x, y)$$

$$G(u, v) = B \text{sinc}(Bu) \delta(v) ** F(u, v)$$

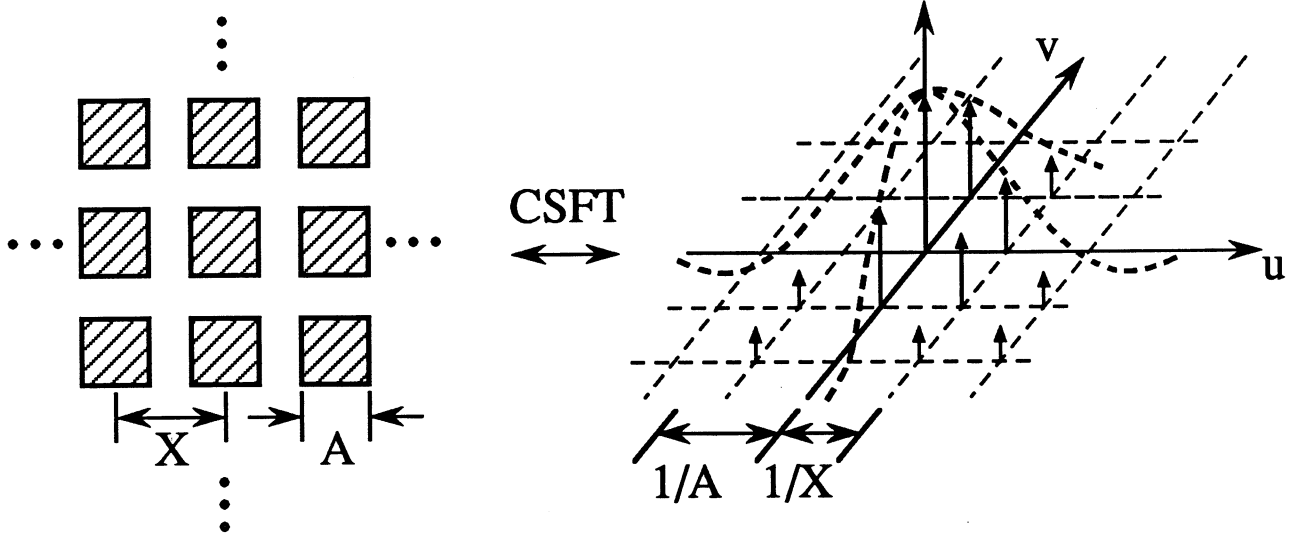
## 2-D Comb and Replication Operators

$$\text{comb}_{XY}[f(x,y)] = \sum_m \sum_n f(mX, nY) \delta(x-mX, y-nY)$$

$$\text{rep}_{XY}[f(x, y)] = \sum_m \sum_n f(x-mX, y-nY)$$

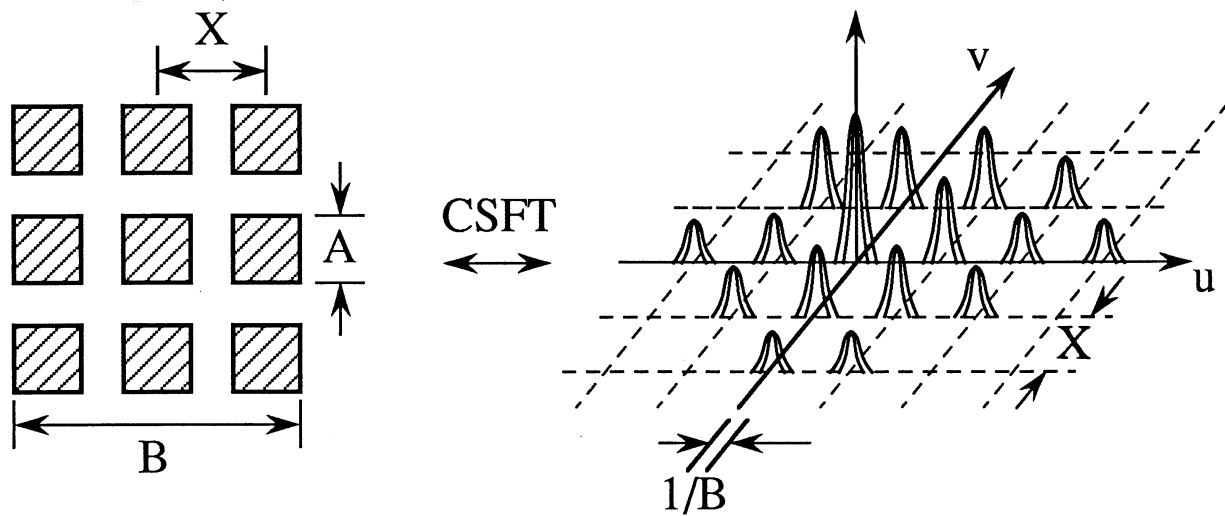
Transform relation

$$\text{rep}_{XY}[f(x,y)] \stackrel{\text{CSFT}}{\leftrightarrow} \frac{1}{XY} \text{comb} \frac{1}{X} \frac{1}{Y} [F(u,v)]$$



$$f(x, y) = \text{rep}_{XX} \left[ \text{rect} \left( \frac{x}{A}, \frac{y}{A} \right) \right]$$

$$F(u, v) = \frac{1}{X^2} \text{comb} \frac{1}{X} \frac{1}{X} [A^2 \text{sinc}(Au, Av)]$$



$$g(x, y) = \text{rect}\left(\frac{x}{B}, \frac{y}{B}\right) f(x, y)$$

$$G(u, v) = B^2 \text{sinc}(Bu, Bv) ** F(u, v)$$

## General Relations Between Space and Frequency Domains

Space Domain	Frequency Domain
spatial lattice	reciprocal lattice
microscopic properties	macroscopic properties
macroscopic properties	microscopic properties