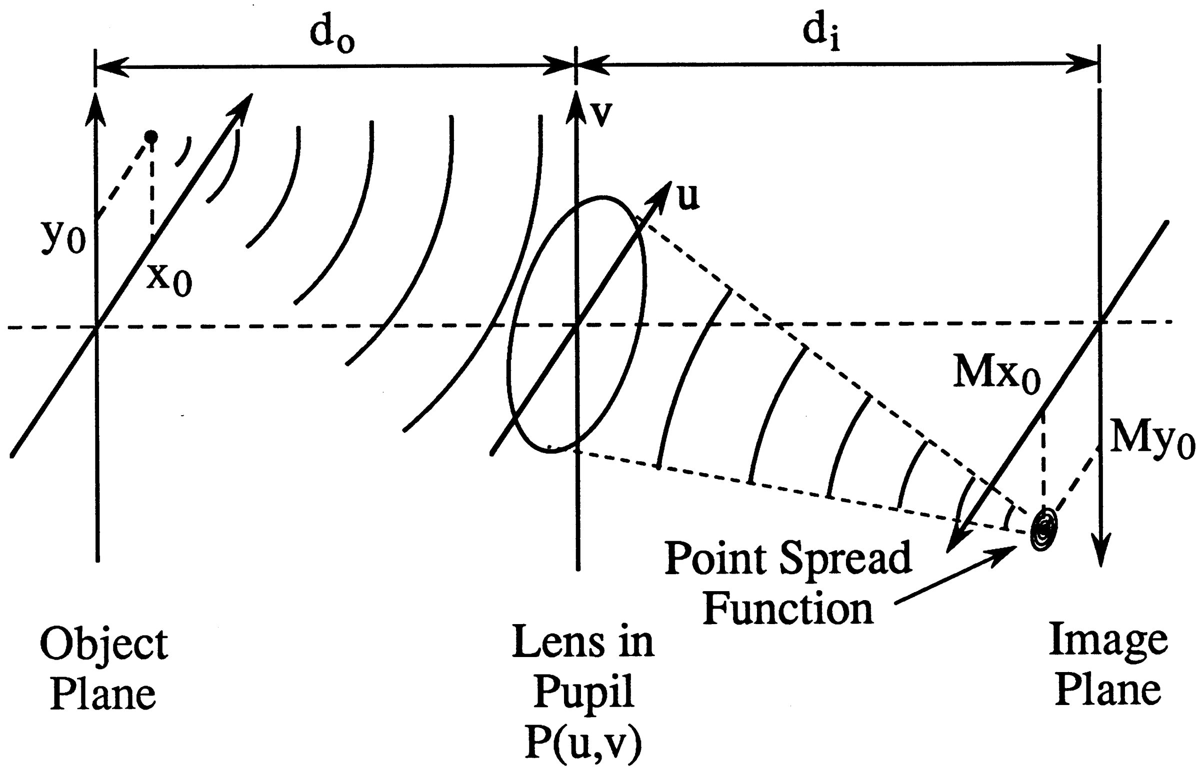
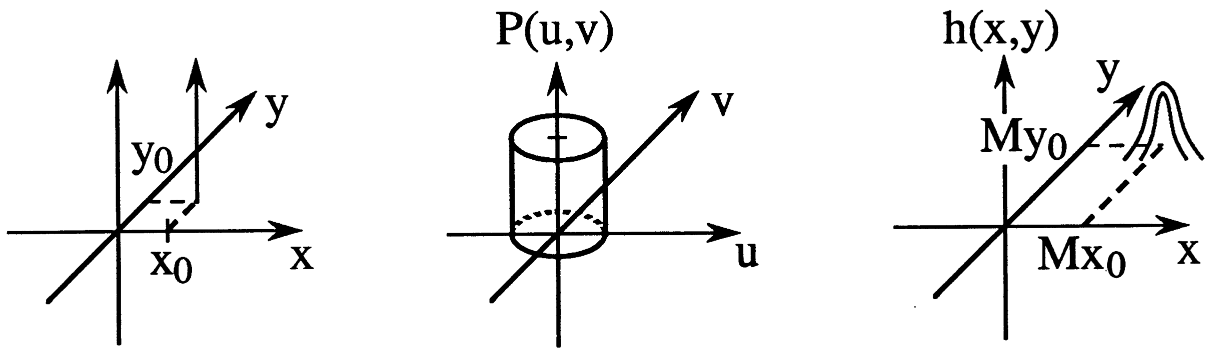
2.1.3 LINEAR, SHIFT-INVARIANT (LSIV) IMAGING SYSTEMS

Imaging a Point Source



$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \qquad M = \frac{d_i}{d_o} \quad \text{magnification}$$

Alternate Representation



Object

Pupil

Image
(Point Spread Function)

Relation Between Pupil and Point Spread

Coherent Imaging System

$$h_C(x,y) \overset{\text{CSFT}}{\leftrightarrow} H_C(u,v) \quad \text{(coherent transfer function)}$$

$$H_C(u,v) = P(\lambda d_i u, \lambda d_i v) \quad (\lambda\text{- wavelength of optical radiation})$$

Incoherent Imaging System

$$h_I(x,y) \overset{\text{CSFT}}{\leftrightarrow} H_I(u,v)$$

$$h_I(x,y) = |h_C(x,y)|^2$$

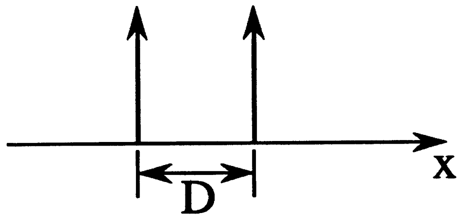
Optical Transfer Function

$$\mathscr{H}(u,v) = H_I(u,v)/H_I(0,0)$$

Modulation Transfer Function (MTF)

$$M(u,v) = |\mathscr{H}(u,v)|$$

Imaging Two Point Sources



Object

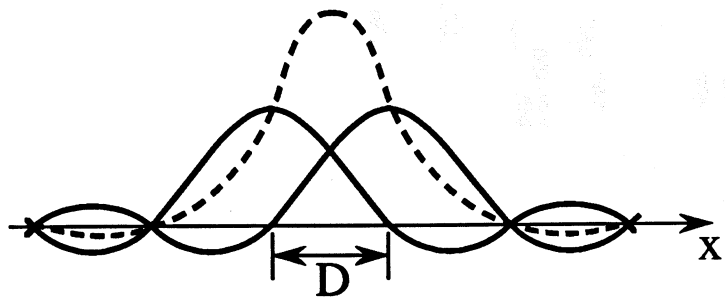
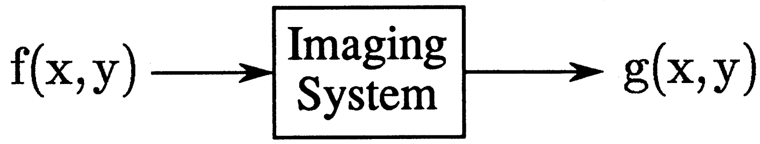


Image of 2 point sources separated by Rayleigh distance

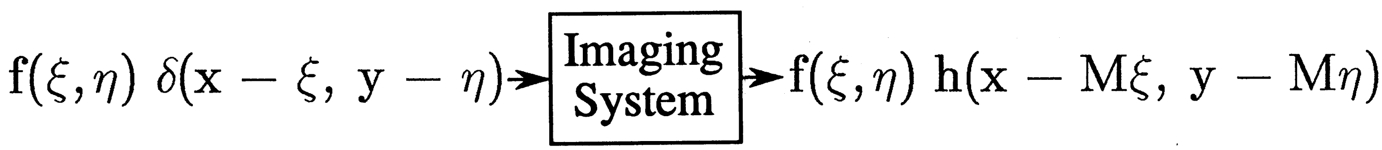
- Other criteria exist for resolution.

- It is possible to modify the transmittance function within the pupil to improve resolution. This is referred to as apodization.

Imaging an Extended Object



$$f(x,y) = \int \int f(\xi,\eta)\ \delta(x - \xi,\ y - \eta)\ d\xi d\eta$$



(by homogeneity)

By superposition,

$$g(x,y) = \int \int f(\xi, \eta)\, h(x - M\xi,\ y - M\eta)\, d\xi d\eta$$

$$g(x,y) = \frac{1}{M^2} \int \int f\left(\frac{\xi}{M}, \frac{\eta}{M}\right) h(x - \xi,\ y - \eta) d\xi d\eta$$

diffraction-limited image	image predicted by geometrical optics	2-D convolution integral

- This type of analysis extends to a very large class of imaging systems.

- Generally, the shape of the point spread function will depend on its position in the image plane. In this case, the image plane is partitional into patches within which the point spread function is approximately the same.

- In what follows, we will always assume unity magnification.

CSFT and LSIV Imaging Systems
Convolution Theorem

As in the 1-D case, we have the following identity for any functions f(x,y) and h(x,y),

$$\int\int f(\xi,\eta)h(x-\xi,y-\eta)d\xi d\eta = \int\int f(x-\xi,y-\eta)h(\xi,\eta)d\xi d\eta$$

Consider the image of a complex exponential object:

$$e^{i2\pi[ux+vy]} \longrightarrow \boxed{\text{Imaging System}} \longrightarrow \int\int e^{i2\pi[u(x-\xi)+v(y-\eta)]}$$

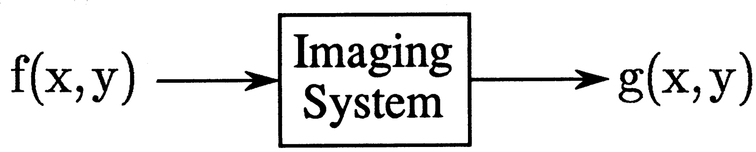
$$\times\ h(\xi,\eta)d\xi d\eta$$

$$= e^{i2\pi[ux+vy]} \int\int h(\xi,\eta)\ e^{-i2\pi[u\xi+v\eta]}\ d\xi d\eta$$

$$= H(u,v)\ e^{i2\pi[ux+vy]}$$

$\Rightarrow\quad e^{i2\pi[ux+vy]}$ is an eigenfunction of the system

Now consider again the extended object:



$$f(x,y) = \int \int F(u,v)\, e^{i2\pi[ux+vy]}\, dudv$$

By linearity:

$$g(x,y) = \int \int H(u,v)\, F(u,v)\, e^{i2\pi[ux+vy]}\, dudv$$

$$\Rightarrow G(u,v) = H(u,v)\, F(u,v)$$

Convolution Theorem

Since f(x,y) and h(x,y) are arbitrary signals, we have the following Fourier transform relation

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(\xi,\eta)\, f_2(x-\xi,\ y-\eta)d\xi d\eta \overset{\text{CSFT}}{\leftrightarrow} F_1(u,v)\, F_2(u,v)$$

or

$$f_1(x,y) ** f_2(x,y) \overset{\text{CSFT}}{\leftrightarrow} F_1(u,v)F_2(u,v)$$

Product Theorem

By reciprocity, we also have the following result

$$f_1(x,y)f_2(x,y) \overset{\text{CSFT}}{\leftrightarrow} F_1(u,v) ** F_2(u,v)$$

As in the 1-D case, this can be very useful for calculating the transforms of certain functions.

Transfer Function of Incoherent Imaging System

Recall

$$h_I(x,y) = |\, h_C(x,y)\, |^2$$

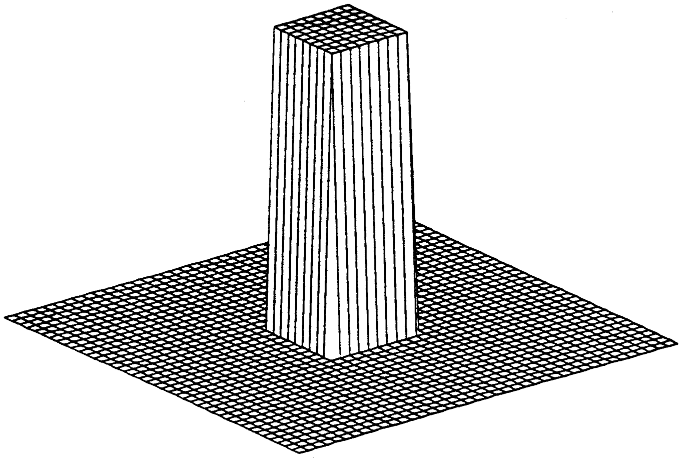
$$= h_C(x,y)\, h_C^*(x,y)$$

$$\mathscr{F}\{h_C^*(x,y)\} = \int \int h_C^*(x,y)\, e^{-j2\pi[ux+vy]}\, dxdy$$

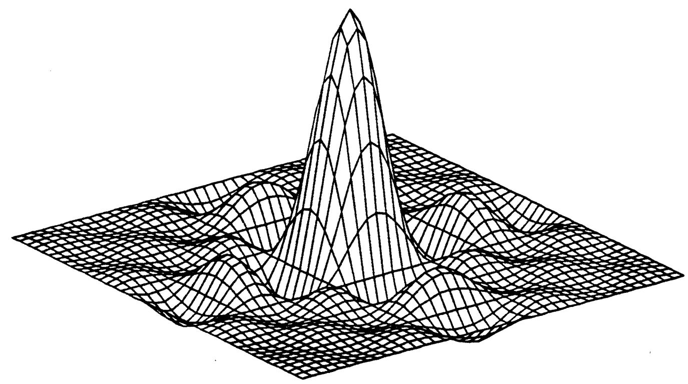
$$= \{\int \int h_C(x,y)\, e^{-j2\pi[(-u)x+(-v)y]}\, dxdy\}^*$$

$$= H_C^*(-u,\ -v)$$

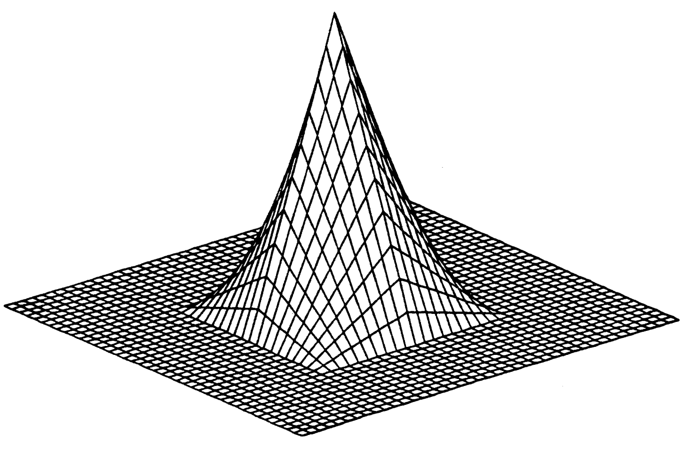
$$\therefore \quad H_I(u,v) = H_C(u,v) ** H_C^*(-u,\ -v)$$



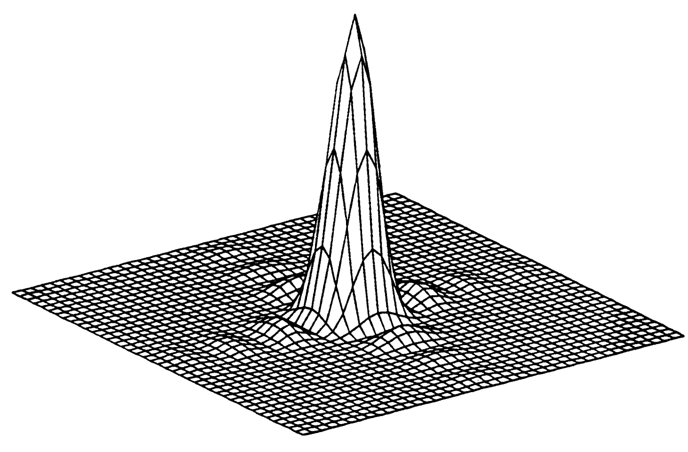
$H_C(u, v)$



$h_C(x, y)$



$H_I(u, v)$



$h_I(x, y)$