Digital Filter Design

Synopsis

- Overview of filter design problem
- Finite impulse response filter design
- Infinite impulse response filter design

Overview

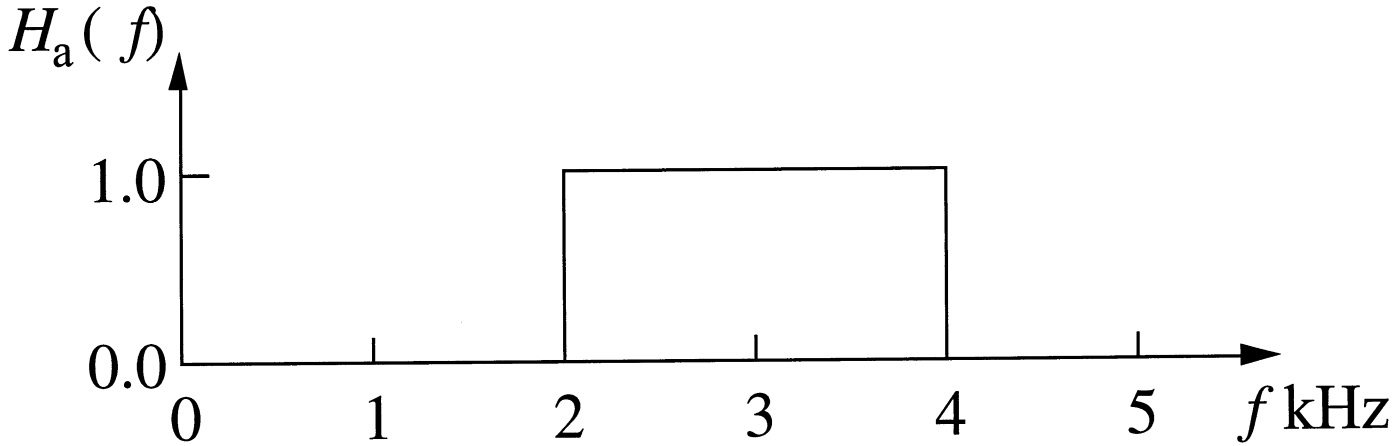
- Filter design problem consists of three tasks
  - specification
  - approximation
  - realization
- Consider a simple example to illustrate these tasks

Filter Design Example

- An analog signal contains frequency components ranging from DC (0 Hz) to 5 kHz
- Design a digital system to remove all but the frequencies in the range 2 - 4 kHz
- Assume signal will be sampled at 10 kHz rate

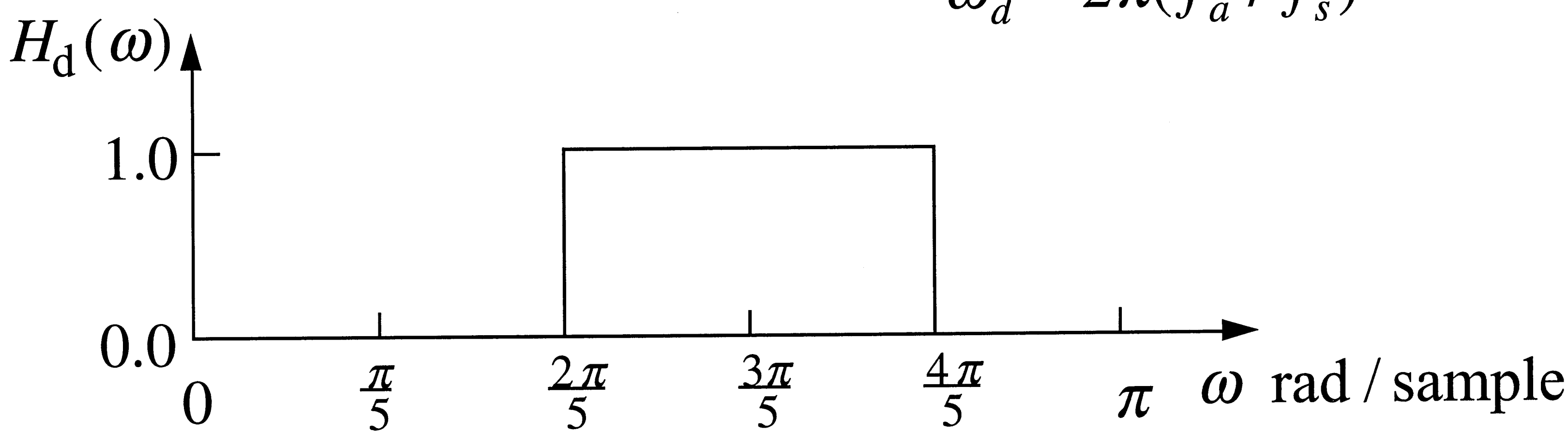
Specification

- **Ideal analog filter**

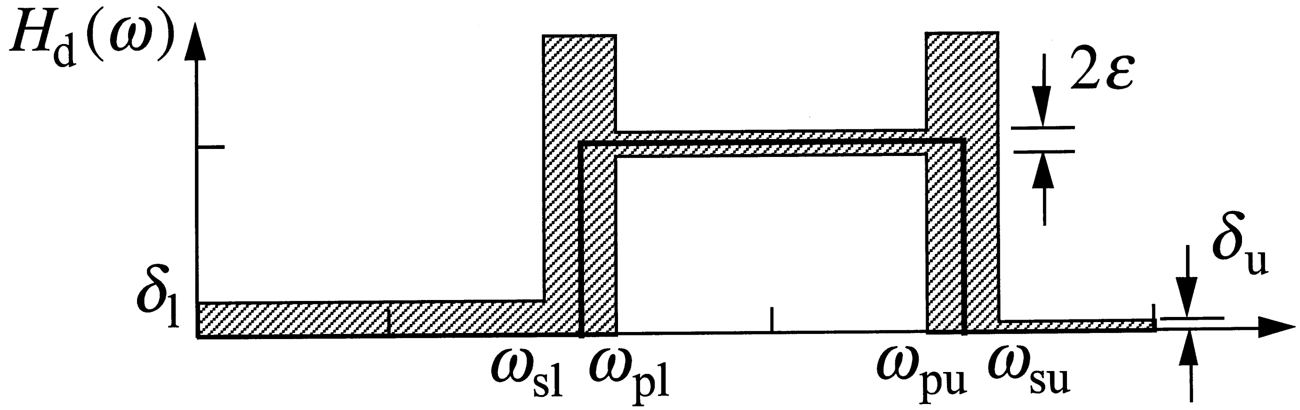


- **Ideal digital filter**

$$\omega_d = 2\pi(f_a / f_s)$$



Filter Tolerance Specification



ω_{pl}, ω_{pu} **- lower and upper passband edges**

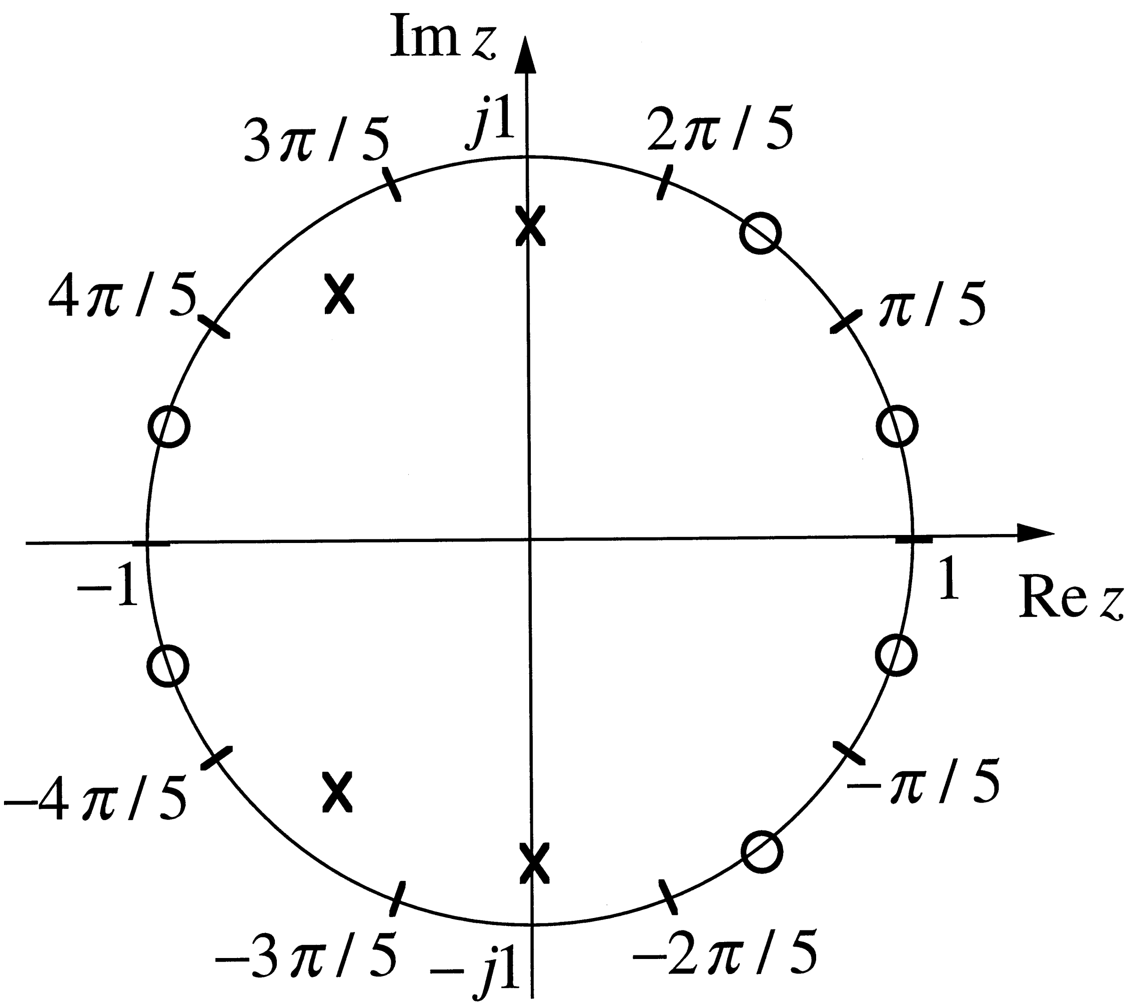
ω_{sl}, ω_{su} **- lower and upper stopband edges**

δ_l, δ_u **- lower and upper stopband ripple**

ε **- passband ripple**

Approximation

- **Design by positioning poles and zeros (PZ plot design)**



Approximation (Cont.)

- **Transfer function for PZ plot filter**

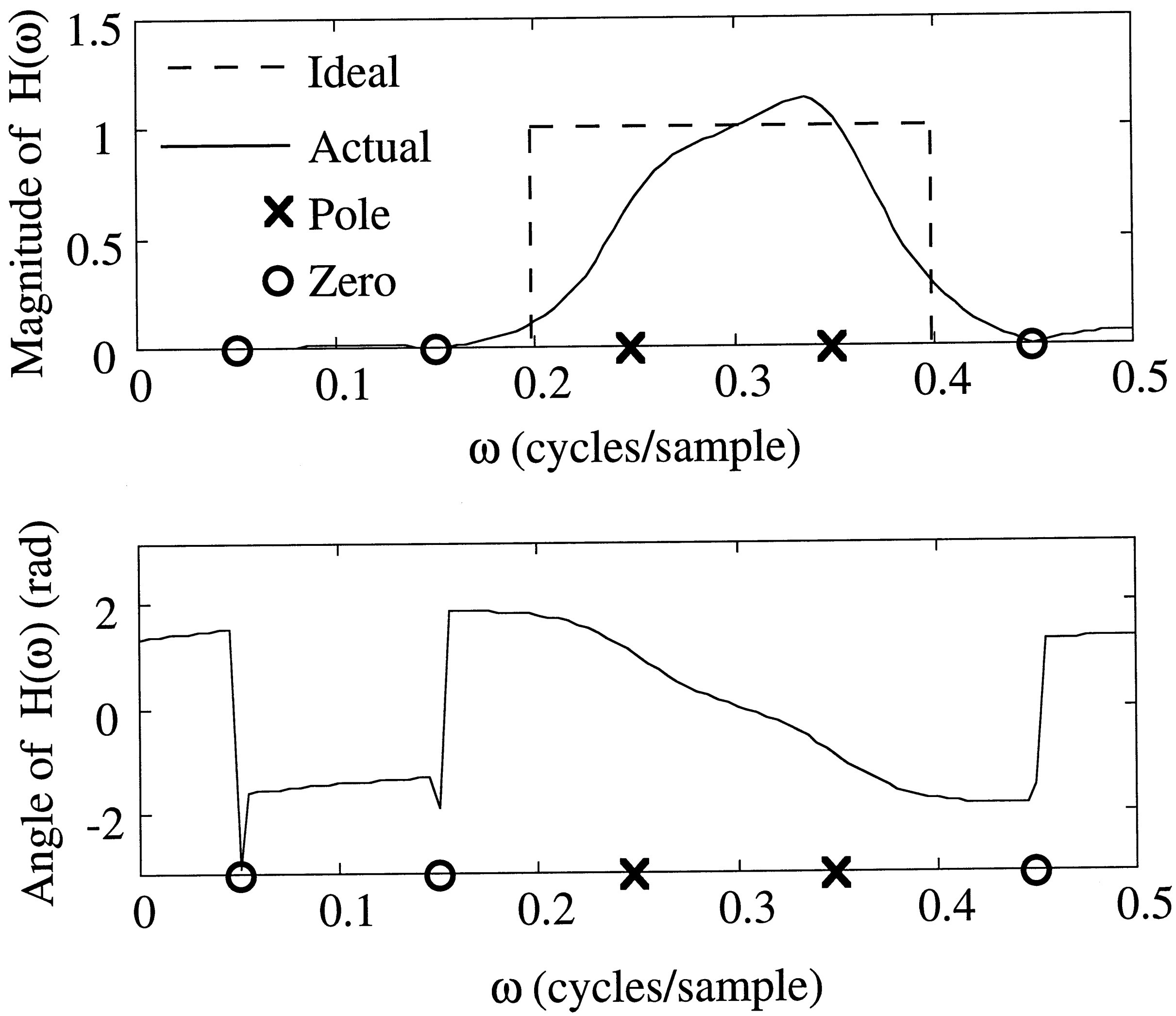
$$H(z) = K\frac{\left(z - e^{j\pi/10}\right)\left(z - e^{j3\pi/10}\right)\left(z - e^{j9\pi/10}\right)}{\left(z - 0.8e^{j5\pi/10}\right)\left(z - 0.8e^{j7\pi/10}\right)}$$

$$\times\frac{\left(z - e^{-j\pi/10}\right)\left(z - e^{-j3\pi/10}\right)\left(z - e^{-j9\pi/10}\right)}{\left(z - 0.8e^{-j5\pi/10}\right)\left(z - 0.8e^{-j7\pi/10}\right)}$$

Choose constant *K* to yield unity magnitude response at center of passband

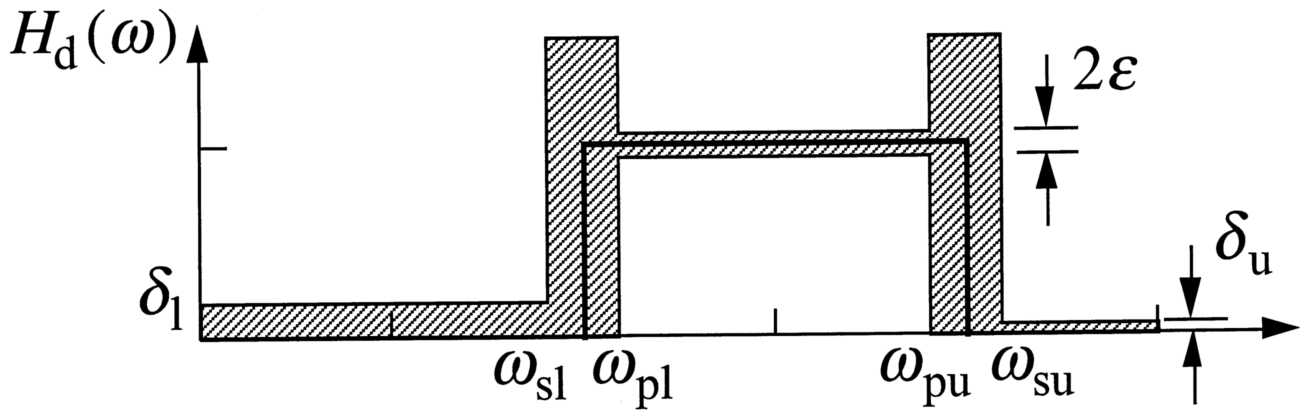
Approximation (Cont.)

- **Frequency response of PZ plot filter**



Approximation (Cont.)

- **Comments on PZ plot design**
  - Passband asymmetry is due to extra zeros in lower stopband
  - Phase is highly nonlinear
  - Zeros on unit circle drive magnitude response to zero and result in phase discontinuities
  - Pole-zero plot yields intuition about filter behavior, but does not suggest how to meet design specifications:



Overview (Cont.)

1. Specification - What frequency response or other characteristics of the filter are desired?
2. Approximation - What are the coefficients or, equivalently, pole and zeros of the filter that will approximate the desired characteristics?
3. Realization - How will the filter be implemented?

Realization

- **Cascade form of transfer function**

$$H(z) = K\frac{\left(z - e^{j\pi/10}\right)\left(z - e^{j3\pi/10}\right)\left(z - e^{j9\pi/10}\right)}{\left(z - 0.8e^{j5\pi/10}\right)\left(z - 0.8e^{j7\pi/10}\right)}$$

$$\times\frac{\left(z - e^{-j\pi/10}\right)\left(z - e^{-j3\pi/10}\right)\left(z - e^{-j9\pi/10}\right)}{\left(z - 0.8e^{-j5\pi/10}\right)\left(z - 0.8e^{-j7\pi/10}\right)}$$

Cascade Form Realization

- **Cascade form in second order sections with real-valued coefficients**

$$H(z) = K\left[z^2 - 2\cos(\pi/10)z + 1\right]$$

$$\times\left[\frac{z^2 - 2\cos(3\pi/10)z + 1}{z^2 - 1.6\cos(5\pi/10)z + 0.64}\right]$$

$$\times\left[\frac{z^2 - 2\cos(9\pi/10)z + 1}{z^2 - 1.6\cos(7\pi/10)z + 0.64}\right]$$

Cascade Form Realization

- **Convert to negative powers of z**

$$H(z) = K\left[1 - 2\cos(\pi/10)z^{-1} + z^{-2}\right]$$

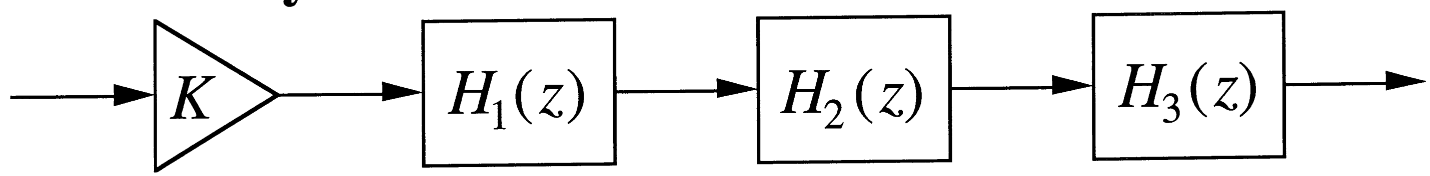
$$\times\left[\frac{1 - 2\cos(3\pi/10)z^{-1} + z^{-2}}{1 - 1.6\cos(5\pi/10)z^{-1} + 0.64z^{-2}}\right]$$

$$\times\left[\frac{1 - 2\cos(9\pi/10)z^{-1} + z^{-2}}{1 - 1.6\cos(7\pi/10)z^{-1} + 0.64z^{-2}}\right]$$

(ignoring overall time advance of 2 sample units)

Cascade Form Realization

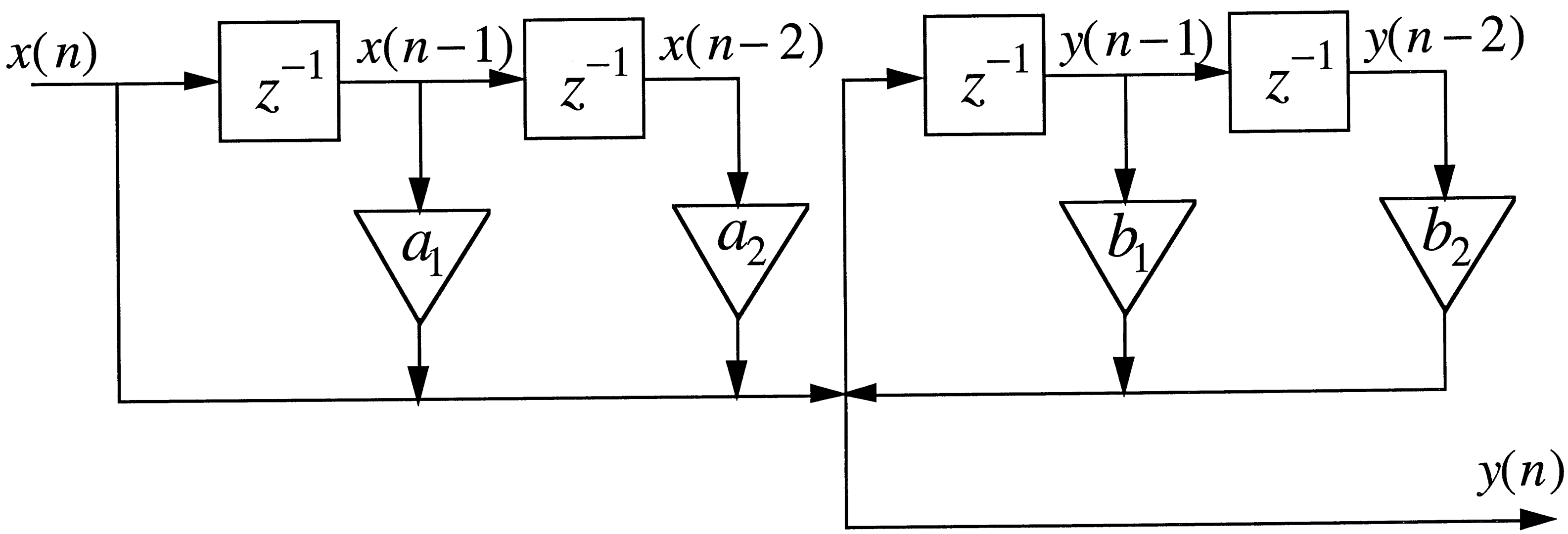
- **Overall system**



- **Second order stages**

$$H_i(z) = \frac{1 + a_{1i}\, z^{-1} + a_{2i}\, z^{-2}}{1 - b_{1i}\, z^{-1} - b_{2i}\, z^{-2}}, \quad i = 1,2,3$$

$$y_i[n] = x_i[n] + a_{1i}\, x_i[n-1] + a_{2i}\, x_i[n-2] + b_{1i}\, y_i[n-1] + b_{2i}\, y_i[n-2]$$



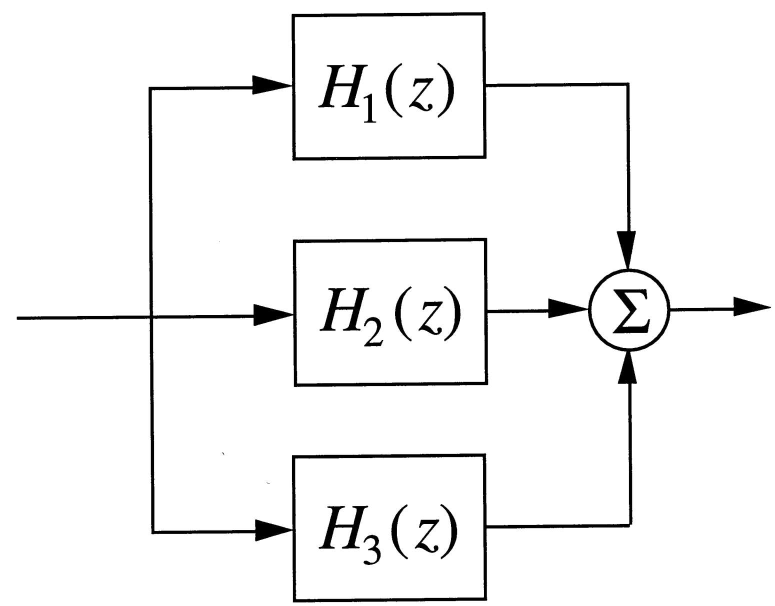
Parallel Form Realization

- **Expand transfer function in partial fractions**

$$H(z) = d_0 + d_1 z^{-1} + d_2 z^{-2}$$

$$+ \frac{a_{01} + a_{11} z^{-1}}{1 - b_{11} z^{-1} - b_{21} z^{-2}}$$

$$+ \frac{a_{02} + a_{12} z^{-1}}{1 - b_{12} z^{-1} - b_{22} z^{-2}}$$



Direct Form Realization

- **Multiply out all factors in numerator and denominator of transfer function**

$$H(z) = \frac{a_0 + a_1 z^{-1} + \cdots + a_6 z^{-6}}{1 - b_1 z^{-1} - \cdots - b_4 z^{-4}}$$