

1.6.2 DFT PROPERTIES AND PAIRS

1. linearity

$$a_1x_1(n) + a_2x_2(n) \xleftrightarrow{\text{DFT}} a_1X_1(k) + a_2X_2(k)$$

2. shifting

$$x(n - n_0) \xleftrightarrow{\text{DFT}} X(k)e^{-j2\pi kn_0/N}$$

3. modulation

$$x(n)e^{j2\pi k_0 n/N} \xleftrightarrow{\text{DFT}} X(k - k_0)$$

4. reciprocity

$$X(n) \xleftrightarrow{\text{DFT}} N x(-k)$$

5. Parseval's relation

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

6. Initial value

$$\sum_{n=0}^{N-1} x(n) = X(0)$$

7. Periodicity

$$x(n + mN) = x(n) \quad \text{for all integers } m$$

$$X(k + \ell N) = X(k) \quad \text{for all integers } \ell$$

8. Relation to DTFT of a finite length sequence.

Let $x_0(n) \neq 0$ only for $0 \leq n \leq N-1$

$$x_0(n) \xleftrightarrow{\text{DTFT}} X_0(e^{j\omega})$$

Define

$$x(n) = \sum_m x_0(n + mN)$$

DFT

$$x(n) \longleftrightarrow X(k)$$

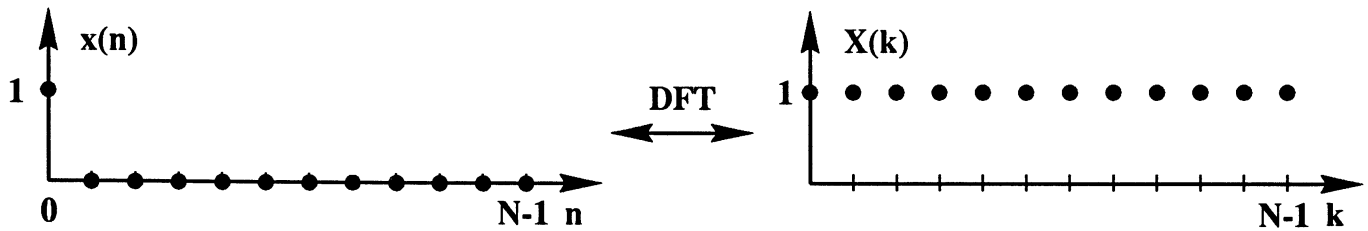
Then

$$X(k) = X_0(e^{j2\pi k/N})$$

Some Transform Pairs

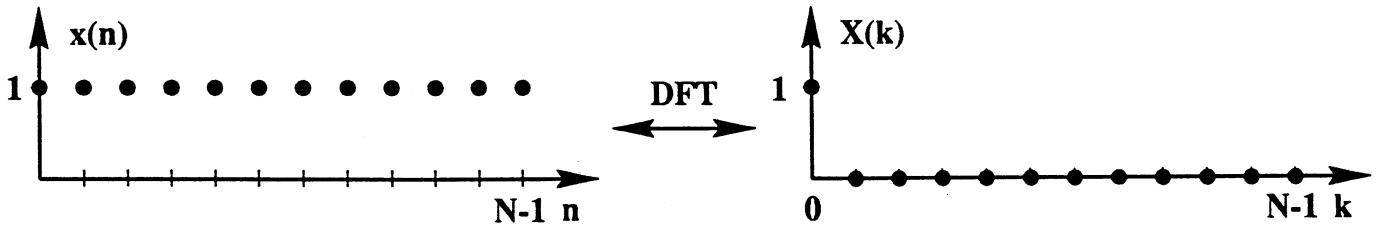
1. $x(n) = \delta(n), \quad 0 \leq n \leq N-1$

$X(k) = 1, \quad 0 \leq k \leq N-1$ (by relation to DTFT)



$$2. \quad x(n) = 1, \quad 0 \leq n \leq N-1$$

$$X(k) = N \delta(k), \quad 0 \leq k \leq N-1 \quad (\text{by reciprocity})$$



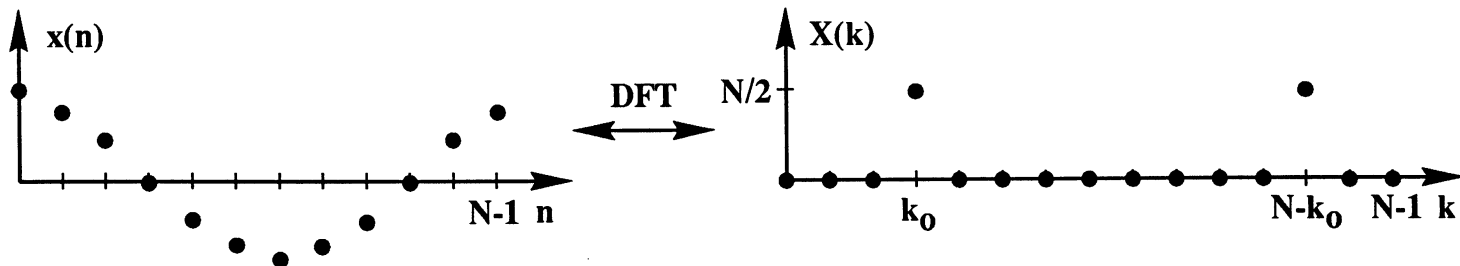
$$3. \quad x(n) = e^{j2\pi k_0 n}, \quad 0 \leq n \leq N-1$$

$$X(k) = N \delta(k - k_0), \quad 0 \leq k \leq N-1$$

(by modulation property)

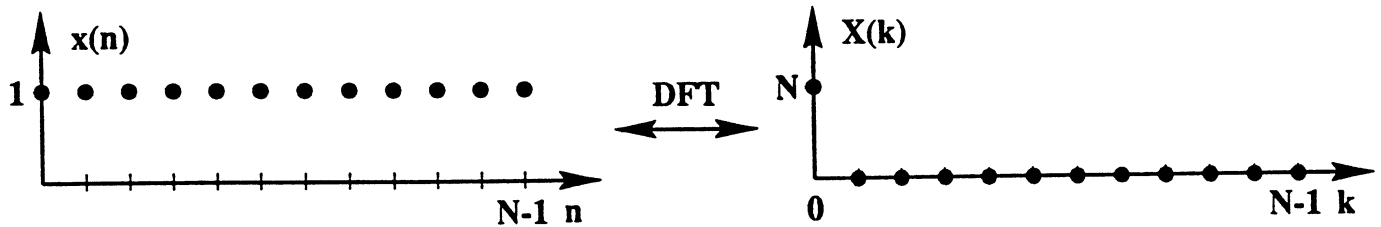
$$4. x(n) = \cos(2\pi k_0 n/N), \quad 0 \leq n \leq N-1$$

$$X(k) = \frac{N}{2} [\delta(k - k_0) + \delta(k - (N - k_0))], \quad 0 \leq k \leq N-1$$



$$2. \quad x(n) = 1, \quad 0 \leq n \leq N-1$$

$$X(k) = N \delta(k), \quad 0 \leq k \leq N-1 \quad (\text{by reciprocity})$$



$$3. \quad x(n) = e^{j2\pi k_0 n}, \quad 0 \leq n \leq N-1$$

$$X(k) = N \delta(k - k_0), \quad 0 \leq k \leq N-1$$

(by modulation property)

$$5. x(n) = \begin{cases} 1, & 0 \leq n \leq M-1 \\ 0, & \text{else} \end{cases} \quad 0 \leq n \leq N-1$$

$$X(k) = e^{-j\frac{2\pi k}{N}(M-1)/2} \frac{\sin[2\pi kM/(2N)]}{\sin[2\pi k/(2N)]}$$

(by relation DTFT)