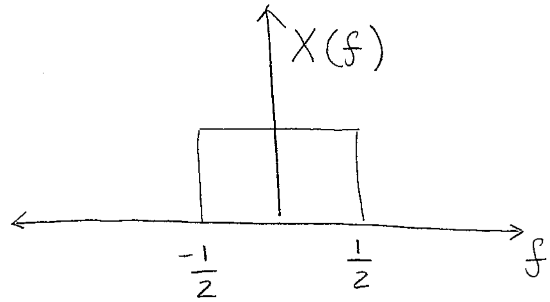
ECE 438 HW#5 Solution Fall 2007

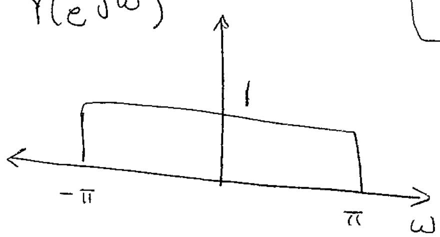
1. $\mathrm{sinc}(t) \overset{CTFT}{\Longleftrightarrow} \mathrm{rect}(-f)$



(i)

(a) $x(t) = \mathrm{sinc}(t)$, $T = 1$

$$\boxed{Y(e^{j\omega}) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right) \text{ for } \omega \in [\pi, \pi]}$$



(b) $x(t) = \mathrm{sinc}(t - 1/4)$, $T = 1$

$$X(f) = \mathrm{rect}(f) e^{-j2\pi f\left(\frac{1}{4}\right)}$$

$$Y(e^{j\omega}) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right) e^{-j2\pi\left(\frac{\omega}{2\pi}\right)\left(\frac{1}{4}\right)} ; \omega \in [-\pi, \pi]$$

$$\boxed{Y(e^{j\omega}) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right) e^{-j\frac{\omega}{4}} \text{ for } \omega \in [-\pi, \pi]}$$

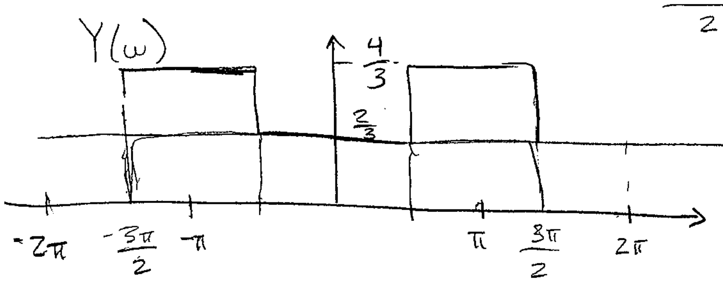
(c) $x(t) = \mathrm{sinc}(t)$, $T = 1/2$

$$y(n) = \mathrm{sinc}\left(\frac{n}{2}\right) = \frac{2\sin\left(\frac{\pi}{2} n\right)}{\pi n}$$

$$\boxed{Y(e^{j\omega}) = 2\,\mathrm{rect}\left(\frac{\omega}{\pi}\right) \text{ for } \omega \in [-\pi, \pi]}$$

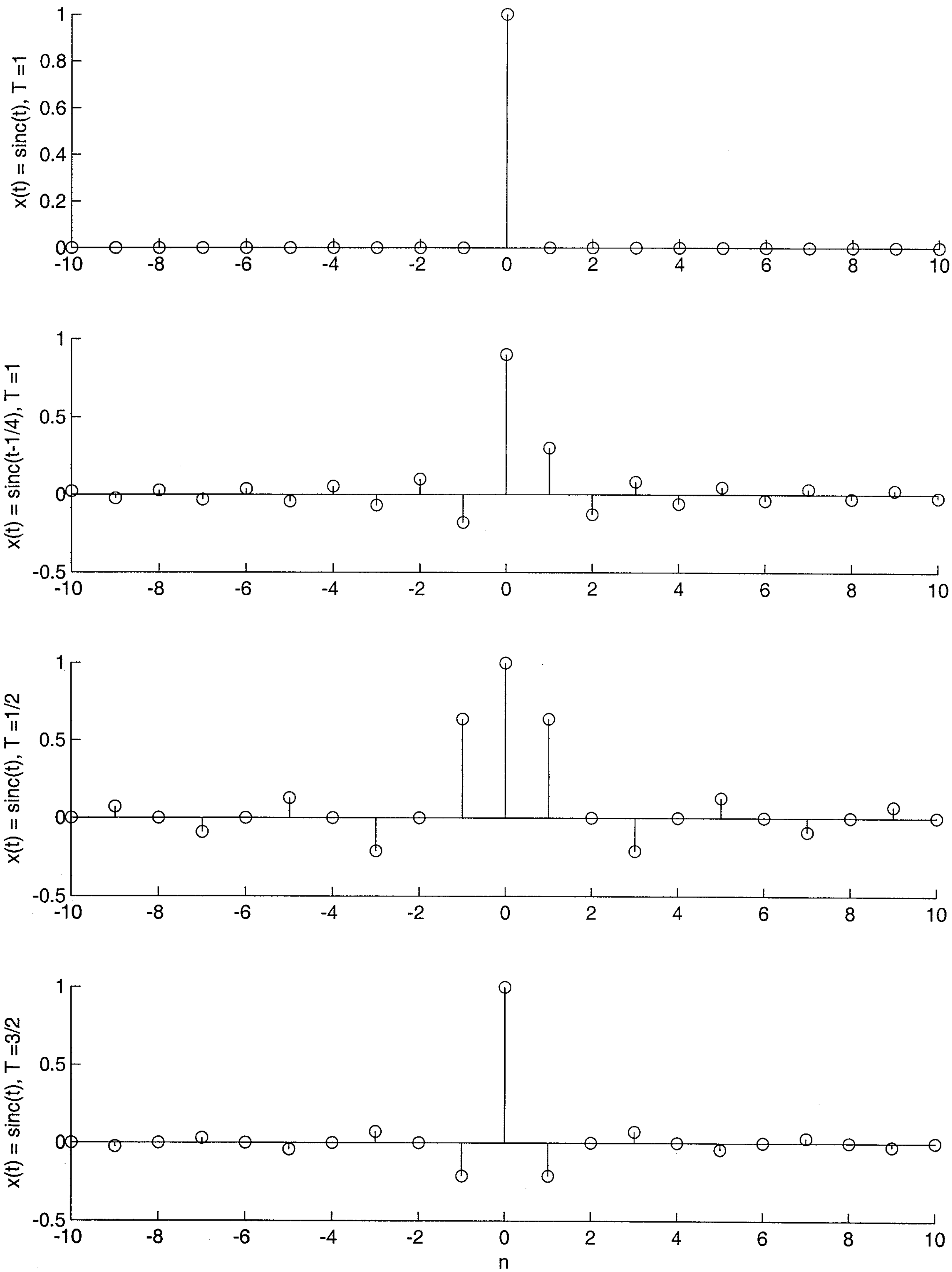
(d) $x(t) = \mathrm{sinc}(t)$, $T = \frac{3}{2}$

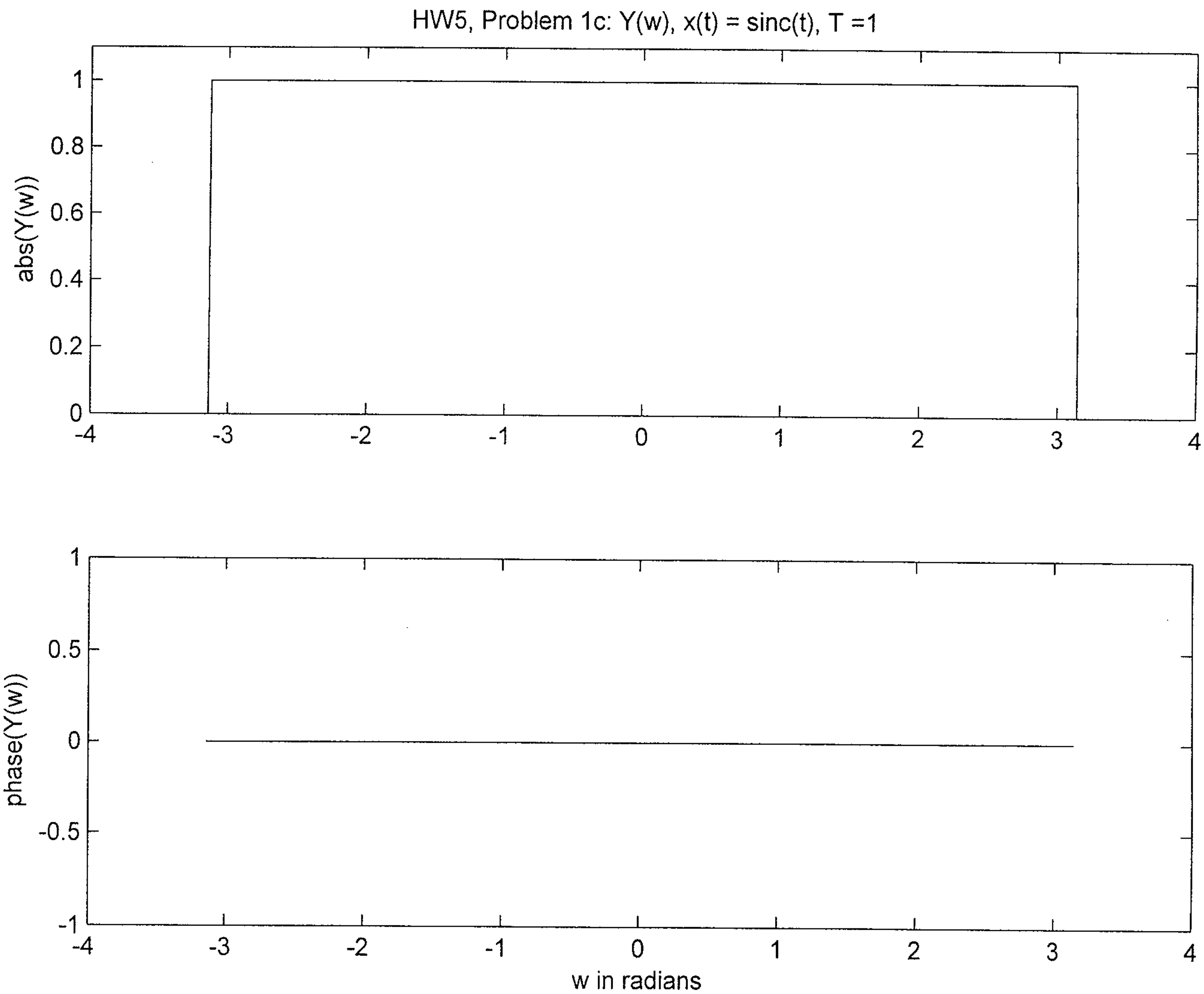
$$y(n) = \mathrm{sinc}\left(\frac{3n}{2}\right) = \frac{\sin \frac{3\pi n}{2}}{\frac{3\pi n}{2}} = \frac{2}{3} \frac{\sin\left(\frac{3\pi}{2} n\right)}{\pi n}$$



$$\boxed{Y(e^{j\omega}) = \begin{cases} \frac{2}{3} & |\omega| \le \frac{\pi}{2} \\ \frac{4}{3} & \frac{\pi}{2} < |\omega| \le \pi \end{cases} \text{ for } \omega \in [-\pi, \pi]}$$

Problem 1b: y(n)




HW5, Problem 1c: Y(w), x(t) = sinc(t), T =1
abs(Y(w))
phase(Y(w))
w in radians
1
0.8
0.6
0.4
0.2
0
0.5
-0.5
-1
-4
-3
-2
-1
2
3
4

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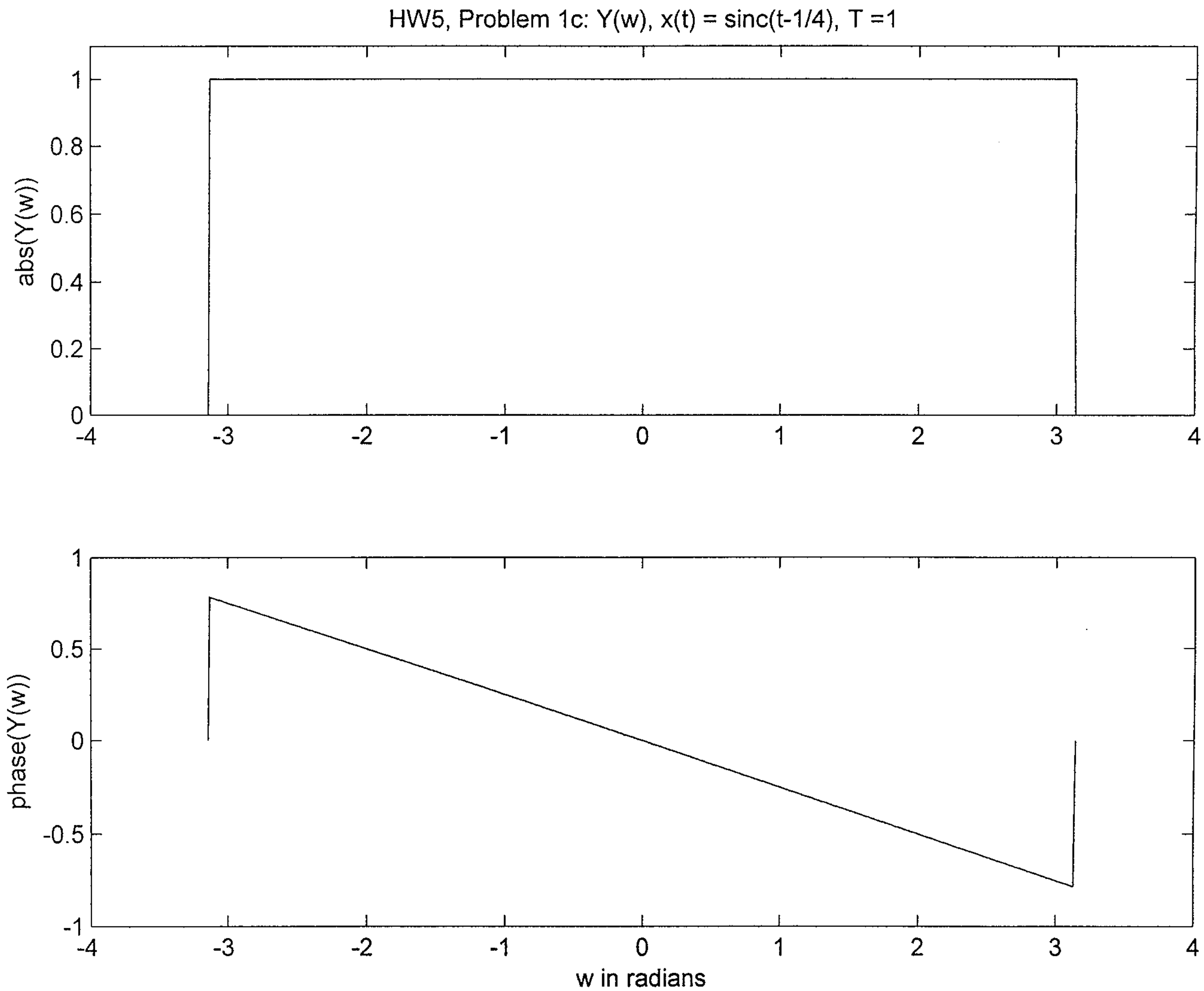

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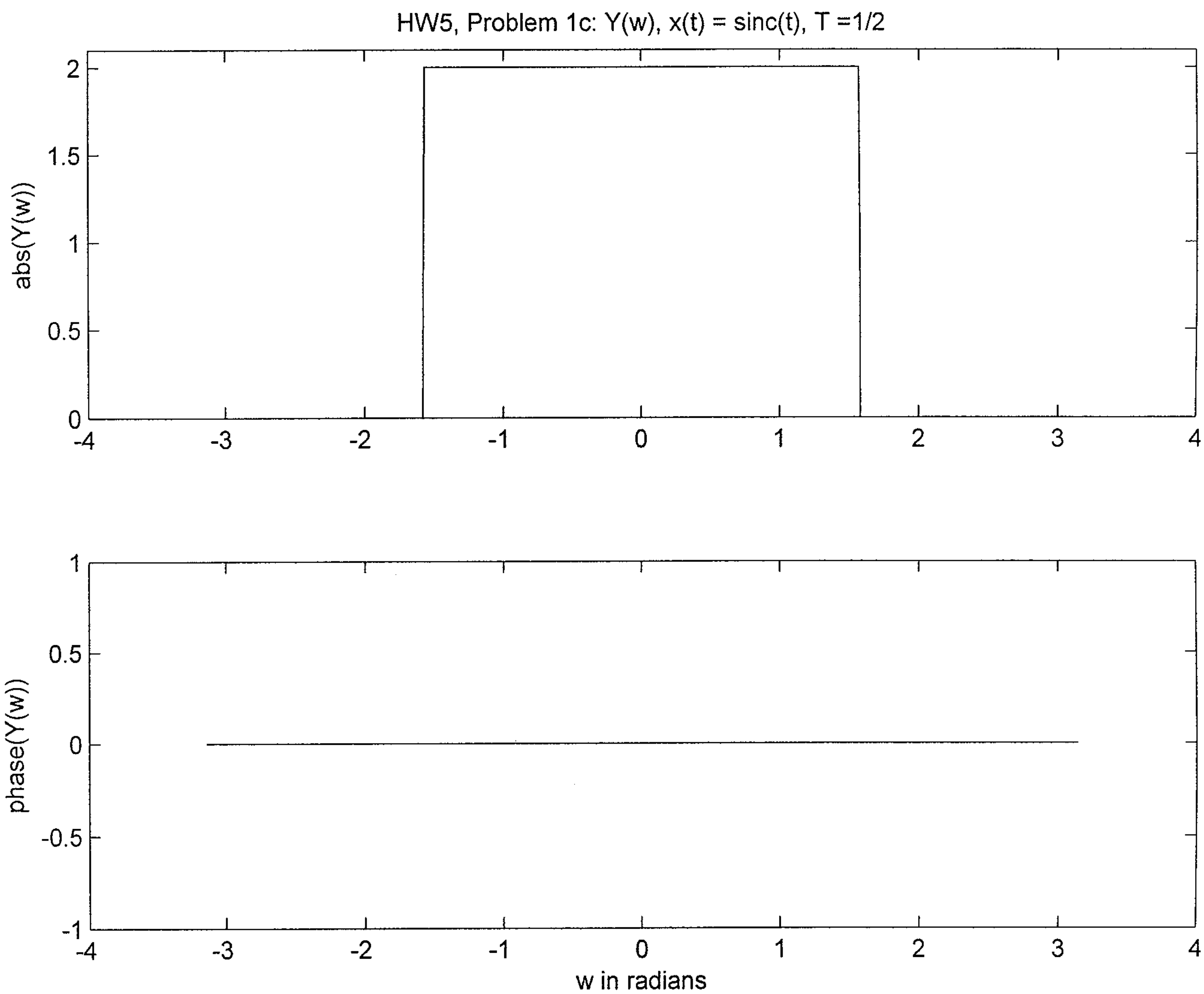
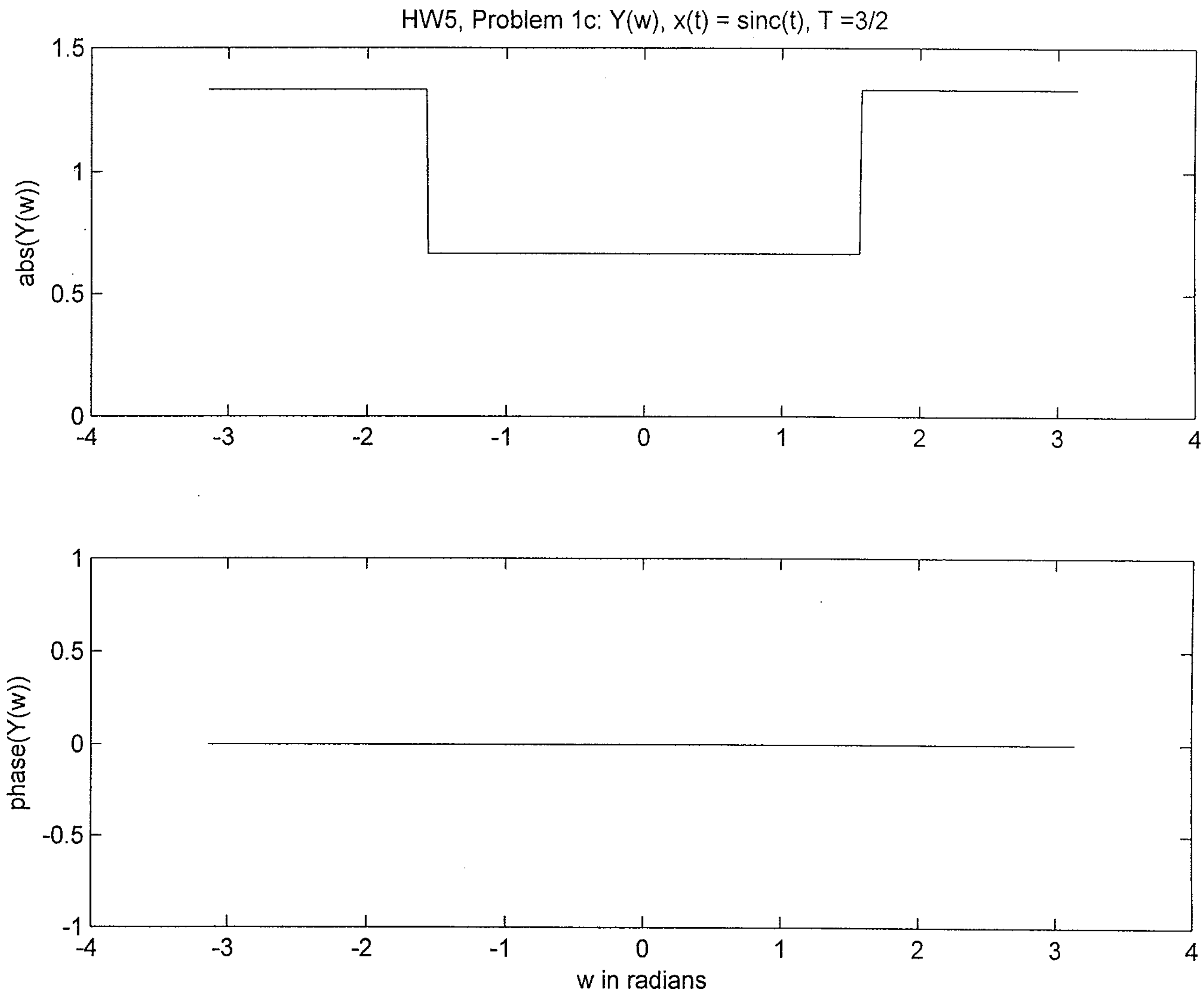




HW5, Problem 1c: Y(w), x(t) = sinc(t), T =3/2
abs(Y(w))
phase(Y(w))
w in radians

2. a) $x_d(n) = \sin\left(\dfrac{2\pi(1\text{KHz})n}{8\text{KHz}}\right) = \sin\left(\dfrac{\pi}{4}n\right)$

$w(n) = x_d(n) * h(n)$ where $h(n) = \text{IDTFT}\{H(e^{j\omega})\}$

Then
$$W(e^{j\omega}) = X_d(e^{j\omega})H(e^{j\omega})$$
$$= \frac{\pi}{j}\left(\delta(\omega - \tfrac{\pi}{4}) - \delta(\omega + \tfrac{\pi}{4})\right)H(e^{j\omega})$$
$$= \frac{\pi}{j}\left(H(e^{j(\frac{\pi}{4})})\delta(\omega - \tfrac{\pi}{4}) - H(e^{j(-\frac{\pi}{4})})\delta(\omega + \tfrac{\pi}{4})\right)$$
$$= \frac{\pi}{j}H(e^{j(\frac{\pi}{4})})\left(\delta(\omega - \tfrac{\pi}{4}) - \delta(\omega + \tfrac{\pi}{4})\right)$$

$$w(n) = H(e^{j(\frac{\pi}{4})})\sin\left(\frac{\pi}{4}n\right) = \frac{3}{4}\sin\left(\frac{\pi}{4}n\right)$$

$$\boxed{y(t) = \frac{3}{4}\sin\left(2\pi(1\text{KHz})t\right)}$$

b) $x_d(n) = \sin\left(\dfrac{2\pi(5\text{KHz})n}{8\text{KHz}}\right) = \sin\left(\dfrac{5\pi}{4}n\right) = \sin\left(\left(\dfrac{5\pi}{4} - 2\pi\right)n\right)$

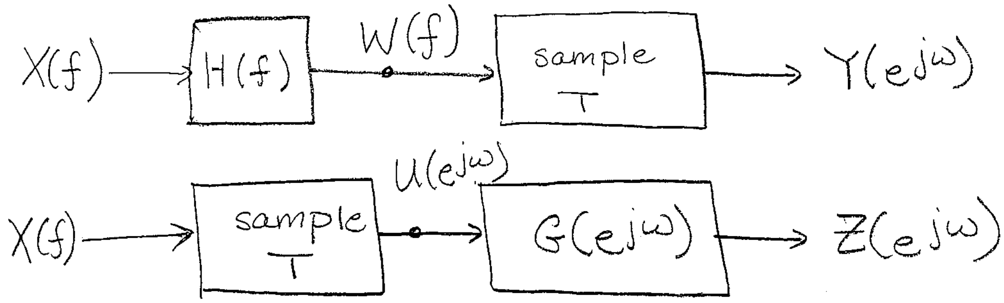
$= \sin\left(\dfrac{-3\pi}{4}n\right)$

$= -\sin\left(\dfrac{3\pi}{4}n\right)$

$$\Rightarrow X_d(e^{j\omega}) = \frac{\pi}{j}\left(\delta(\omega + \tfrac{3\pi}{4}) - \delta(\omega - \tfrac{3\pi}{4})\right)$$
$$w(n) = -H(e^{j(\frac{3\pi}{4})})\sin\left(\frac{3\pi}{4}n\right)$$
$$w(n) = -\frac{1}{4}\sin\left(\frac{3\pi}{4}n\right)$$

$$\boxed{y(t) = \frac{-1}{4}\sin\left(2\pi(3\text{KHz})t\right)}$$

3.



a) $W(f) = X(f)H(f)$

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{l=-\infty}^{\infty} W\left(\frac{\omega - 2\pi l}{2\pi T}\right)$$

$$\boxed{Y(e^{j\omega}) = \frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right) H\left(\frac{\omega - 2\pi l}{2\pi T}\right)}$$

$$U(e^{j\omega}) = \frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right)$$

$$Z(e^{j\omega}) = U(e^{j\omega})\,G(e^{j\omega})$$

$$\boxed{Z(e^{j\omega}) = \left(\frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right)\right) G(e^{j\omega})}$$

b) $$G(e^{j\omega}) = \sum_{K=-\infty}^{\infty} H\left(\frac{\omega - 2\pi K}{2\pi T}\right)$$

$$\Rightarrow Z(e^{j\omega}) = \left(\frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right)\right) G(e^{j\omega})$$

$$= \frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right) \sum_{K=-\infty}^{\infty} H\left(\frac{\omega - 2\pi K}{2\pi T}\right)$$

$$= \frac{1}{T}\sum_{l=-\infty}^{\infty}\sum_{K=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right) H\left(\frac{\omega - 2\pi K}{2\pi T}\right)$$

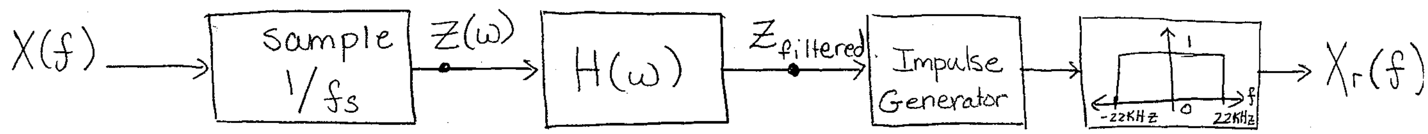
$$= \frac{1}{T}\sum_{l=-\infty}^{\infty} X\left(\frac{\omega - 2\pi l}{2\pi T}\right) H\left(\frac{\omega - 2\pi l}{2\pi T}\right)$$ since

both $X(f)$ and $H(f)$ are bandlimited to $\frac{1}{2T}$.

Thus, $Z(e^{j\omega}) = Y(e^{j\omega})$ when $G(e^{j\omega}) = \sum_{K=-\infty}^{\infty} H\left(\frac{\omega - 2\pi K}{2\pi T}\right)$.

3. c) The second system allows the filtering to be done using DSP. Thus, you can avoid building the analog filter with circuit components. The lower number of parts leads to lower noise, lower cost, and usually lower frustration during implementation.

4.



(a) We want $\dfrac{X_r(f)}{X(f)} = e^{-j2\pi fd}$

Note: $X(f)$ is the band limited input signal and $f_s = 44KHz$

$$Z(\omega) = f_s \sum_{K=-\infty}^{\infty} X\left(\frac{f_s(\omega - 2\pi K)}{2\pi}\right)$$

$$Z_{filtered}(\omega) = Z(\omega)H(\omega)$$

$$= f_s \sum_{K=-\infty}^{\infty} X\left(\frac{f_s(\omega - 2\pi K)}{2\pi}\right) H(\omega)$$

$$X_r(f) = Z_{filtered}\left(\frac{2\pi f}{f_s}\right) \text{rect}\left(\frac{f}{f_s}\right)$$

$$= f_s \sum_{K=-\infty}^{\infty} X\left(\frac{f_s\left(\frac{2\pi f}{f_s} - 2\pi K\right)}{2\pi}\right) H\left(\frac{2\pi f}{f_s}\right) \text{rect}\left(\frac{f}{f_s}\right)$$

$$= f_s X(f) H\left(\frac{2\pi f}{f_s}\right) \text{rect}\left(\frac{f}{f_s}\right)$$

$$\Rightarrow \frac{X_r(f)}{X(f)} = f_s H\left(\frac{2\pi f}{f_s}\right) \text{rect}\left(\frac{f}{f_s}\right) = e^{-j2\pi fd} \text{rect}\left(\frac{f}{f_s}\right)$$

$$\Rightarrow H\left(\frac{2\pi f}{f_s}\right) \text{rect}\left(\frac{f}{f_s}\right) = \frac{1}{f_s} e^{-j2\pi fd} \text{rect}\left(\frac{f}{f_s}\right)$$

and
$$H(\omega) = \frac{1}{f_s} \sum_{K=-\infty}^{\infty} e^{-j2\pi\left(\frac{f_s(\omega - 2\pi K)}{2\pi}\right)d} \text{rect}\left(\frac{1}{f_s}\left(\frac{f_s(\omega - 2\pi K)}{2\pi}\right)\right)$$

$$\boxed{H(\omega) = \frac{1}{f_s} \sum_{K=-\infty}^{\infty} e^{-jf_s(\omega - 2\pi K)d} \text{rect}\left(\frac{\omega - 2\pi K}{2\pi}\right)}$$

or

$$\boxed{H(\omega) = \frac{1}{44KHz} \sum_{K=-\infty}^{\infty} e^{-j(44KHz)(\omega - 2\pi K)d} \text{rect}\left(\frac{\omega - 2\pi K}{2\pi}\right)}$$

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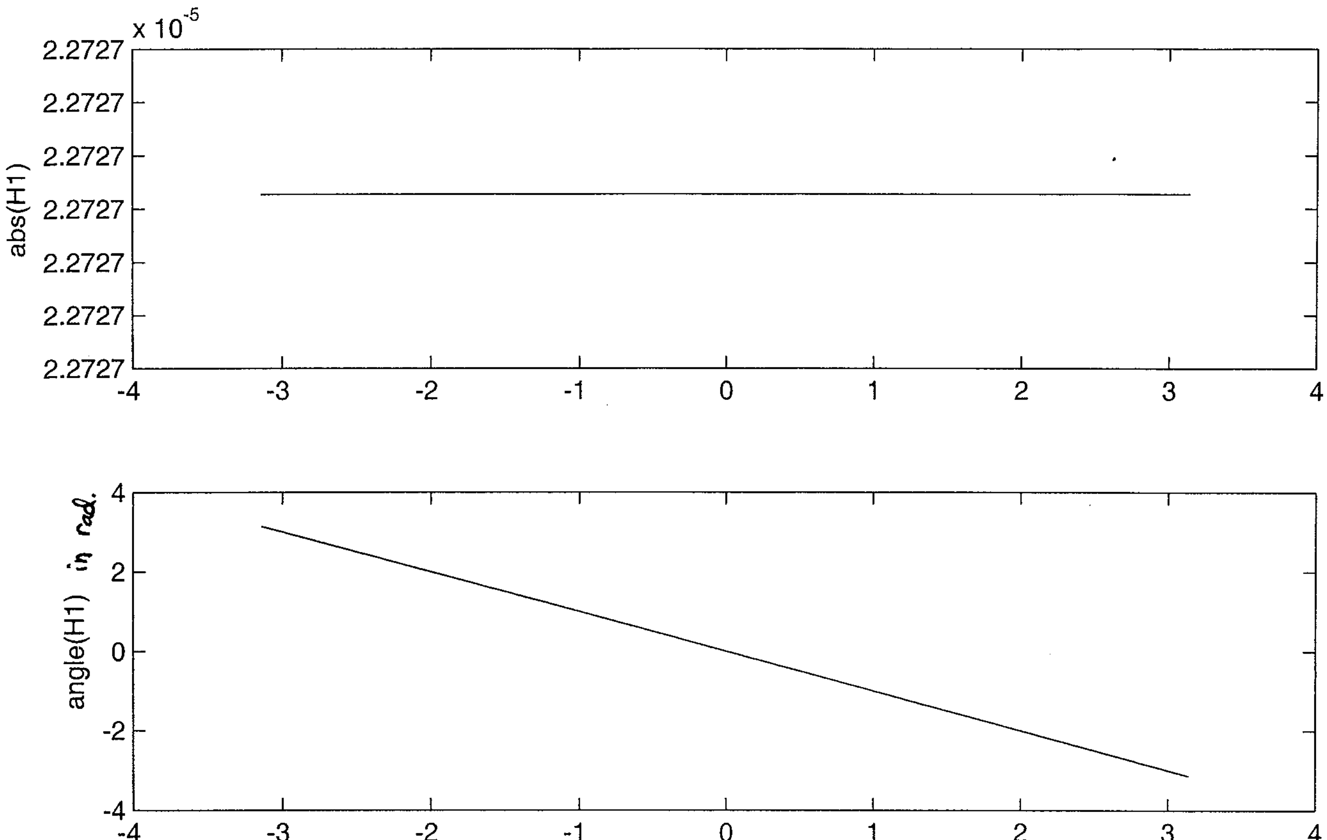
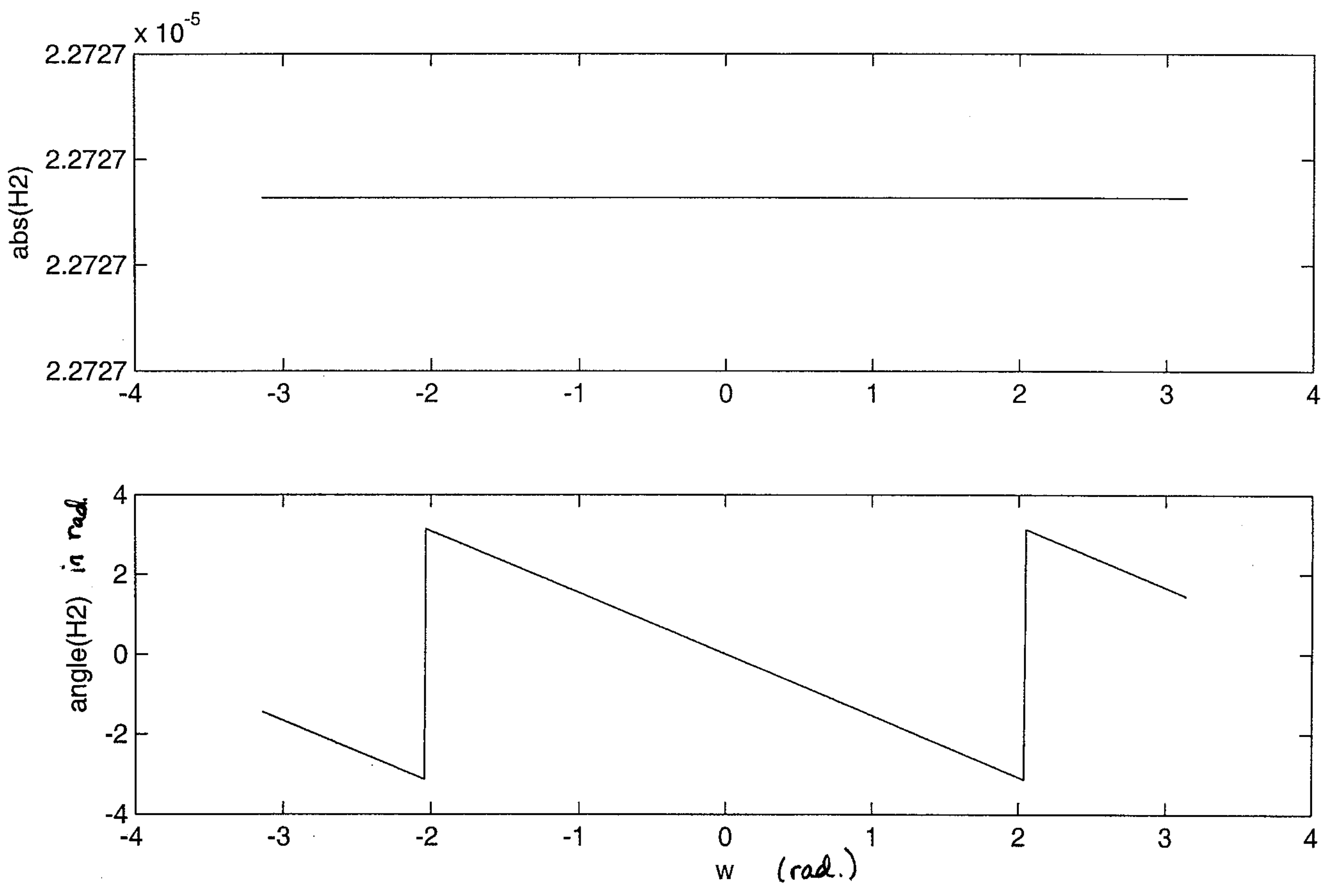
HW5, Problem 4b d = 22.727 usec



HW5, Problem 4b d = 35 usec



4. (c) $h(n) = ?$

$$\alpha = \frac{1}{44KHz} \Rightarrow H(\omega) = \frac{1}{44KHz} \sum_{K=-\infty}^{\infty} e^{-j(\omega - 2\pi K)} \text{rect}\left(\frac{\omega - 2\pi K}{2\pi}\right)$$

So for $\omega \in [-\pi, \pi]$ $\quad H(\omega) = \frac{1}{44KHz} e^{-j\omega} \text{rect}\left(\frac{\omega}{2\pi}\right)$

$$e^{-j\omega} \xrightarrow{\text{IDTFT}} \delta(n-1) \qquad \text{rect}\left(\frac{\omega}{2\pi}\right) \xrightarrow{\text{IDTFT}} \frac{\sin \pi n}{\pi n} = \delta(n)$$

$$h(n) = \frac{1}{44KHz} \delta(n-1) * \delta(n)$$

$$\boxed{h(n) = \frac{1}{44KHz} \delta(n-1)}$$