HW#3 Solutions

1. $x[n] = \cos \omega_0 n$
$y[n] = ?$

$$y[n] = \text{IDTFT}\{H(e^{j\omega})X(e^{j\omega})\}$$
$$X(e^{j\omega}) = \text{DTFT}\{\cos\omega_0 n\}$$
$$= \tfrac{1}{2}[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$$

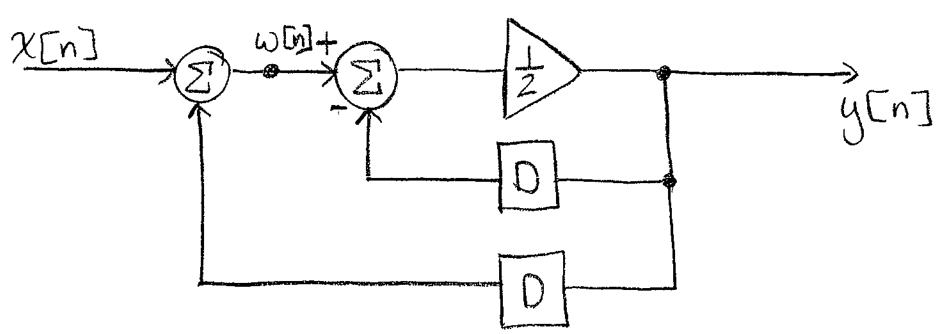
$$y[n] = \text{IDTFT}\{H(e^{j\omega})X(e^{j\omega})\}$$
$$= \text{IDTFT}\{H(e^{j\omega})(\tfrac{1}{2})(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))\}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})(\tfrac{1}{2})(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))e^{j\omega n}\,d\omega$$

$$\boxed{y[n] = \frac{1}{4\pi}\left(H(e^{-j\omega_0})e^{-j\omega_0 n} + H(e^{j\omega_0})e^{j\omega_0 n}\right)}$$

Similarly, for $x[n] = \sin\omega_0 n = \frac{1}{j2}(e^{j\omega_0 n} - e^{-j\omega_0 n})$

$$\boxed{y[n] = \frac{1}{j4\pi}\left(H(e^{j\omega_0})e^{j\omega_0 n} - H(e^{-j\omega_0})e^{-j\omega_0 n}\right)}$$

2. a.



$$w[n] = x[n] + y[n-1]$$
$$y[n] = \tfrac{1}{2}(w[n] - y[n-1])$$
$$y[n] = \tfrac{1}{2}(x[n] + y[n-1] - y[n-1])$$
$$\boxed{y[n] = \tfrac{1}{2}x[n]}$$

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2\. b. $y[n] = (x[n] + y[n-1]) * h_0[n]$,
where $h_0[n] = \text{IDTFT}\{H_0(\omega)\}$

$$\Rightarrow \quad Y(\omega) = (X(\omega) + Y(\omega)e^{-j\omega})H_0(\omega)$$
$$Y(\omega)(1 - e^{-j\omega}H_0(\omega)) = X(\omega)H_0(\omega)$$

$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{H_0(\omega)}{1 - e^{-j\omega}H_0(\omega)}}$$

c\. From part a, $y[n] = \frac{1}{2}x[n]$

$$\Rightarrow \quad Y(\omega) = \frac{1}{2}X(\omega)$$
$$\boxed{H(\omega) = \frac{1}{2}}$$

From part b,

$$H(\omega) = \frac{H_0(\omega)}{1 - e^{-j\omega}H_0(\omega)}, \quad H_0(\omega) = ?$$

$$y_0[n] = \frac{1}{2}(x_0[n] - y_0[n-1])$$
$$Y_0(\omega) = \frac{1}{2}(X_0(\omega) - Y_0(\omega)e^{-j\omega})$$
$$Y_0(\omega)(2 + e^{-j\omega}) = X_0(\omega)$$
$$H_0(\omega) = \frac{Y_0(\omega)}{X_0(\omega)} = \frac{1}{2+e^{-j\omega}}$$

$$H(\omega) = \frac{\dfrac{1}{2+e^{-j\omega}}}{1 - \dfrac{e^{-j\omega}}{2+e^{-j\omega}}} = \frac{1}{2+e^{-j\omega}} \cdot \frac{1}{\dfrac{2+e^{-j\omega} - e^{-j\omega}}{2+e^{-j\omega}}}$$

$$\boxed{H(\omega) = \frac{1}{2}}$$

$\therefore$ The two approaches lead to the same answer.