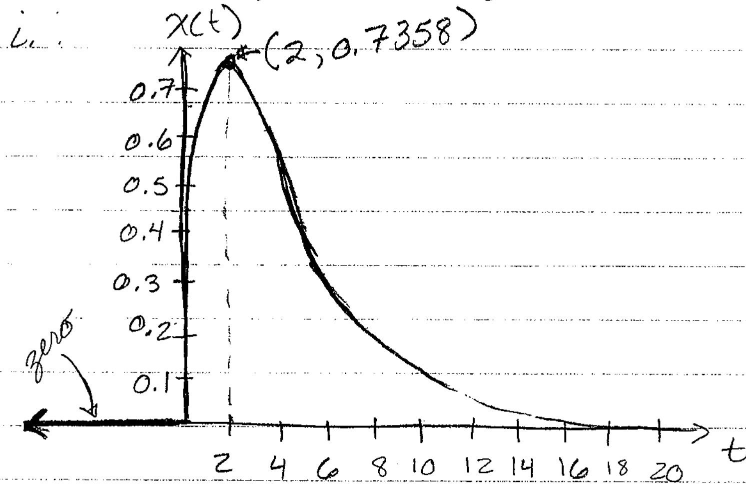
Homework #1 Solutions

1.a. $x(t) = te^{-t/2}u(t)$

i.



ii. right-sided

iii. causal

iv.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |te^{-t/2}u(t)|^2 dt$$

$$= \int_0^t t^2 e^{-t} dt \quad = (-t^2 - 2t - 2)e^{-t}\Big|_{t=0}^{\infty}$$

$$\boxed{E_x = 2}$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |te^{-t/2}u(t)| dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_0^T t^2 e^{-t} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} (-t^2 - 2t - 2)e^{-t}\Big|_{t=0}^{T} = \lim_{T\to\infty} \frac{(-T^2 - 2T - 2)e^{-T}}{2T} + \frac{2}{2T}$$

$$= \lim_{T\to\infty} -\frac{T}{2}e^{-T} - e^{-T} - \frac{e^{-T}}{T} + \frac{2}{2T}$$

$$\boxed{P_x = 0}$$

Homework # 1 Solutions

1. a. iv.

$$x_{rms} = \sqrt{P_x}$$
$$= \sqrt{0}$$
$$\boxed{x_{rms} = 0}$$

$$M_x = \max(x(t))$$

Since $x(t)$ is differentiable over the interval $(0, \infty)$, we can find the points at which $\frac{d}{dt}x(t) = 0$ and

use these points to determine the maximum of $x(t)$.

$$\frac{d}{dt}x(t)\Big|_{t>0} = e^{-t/2} - \frac{t}{2}e^{-t/2}$$

$$\frac{d}{dt}x(t)\Big|_{t>0} = 0 \quad \text{occurs when}$$

$$e^{-t/2} - \frac{t}{2}e^{-t/2} = 0$$

$$e^{-t/2} = \frac{t}{2}e^{-t/2}$$

$$t = 2$$

$$\frac{d^2}{dt^2}x(t)\Big|_{t=2} = \frac{-3}{2}e^{-t/2} + \frac{t}{4}e^{-t/2}\Big|_{t=2}$$

$$= \frac{-3}{2}e^{-1} + \frac{1}{2}e^{-1} = -e^{-1} < 0$$

$$M_x = \max(x(t)) = x(2) = 2e^{-1} \approx 0.7358$$

$$\boxed{M_x \approx 0.7358}$$

1\. a. iv. $A_x = \int_{-\infty}^{\infty} x(t)\,dt$

$$= \int_{-\infty}^{\infty} t e^{-t/2} u(t)\,dt$$

$$= \int_{0}^{\infty} t e^{-t/2}\,dt = (-2t-4)e^{-t/2}\Big|_{t=0}^{\infty}$$

$$\boxed{A_x = 4}$$

$$x_{avg} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} t e^{-t/2} u(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} t e^{-t/2}\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left(-2t-4\right)e^{-t/2}\Big|_{t=0}^{T}$$

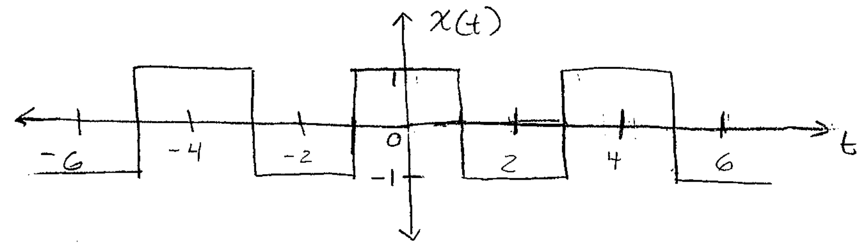
$$= \lim_{T\to\infty} -e^{-T/2} - \frac{2}{T}e^{-T/2} + \frac{2}{T}$$

$$\boxed{x_{avg} = 0}$$

Homework #1 Solutions

1.b. $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \, rect(t/2 - 2k)$

i.



ii. two-sided

iii. neither causal nor anticausal

iv.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \left| \sum_{k=-\infty}^{\infty} (-1)^k rect(t/2 - 2k) \right|^2 = \int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} |(-1)^k rect(t/2 - 2k)|, \right.$$

$$= \int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} |(-1)^k rect(t/2 - 2k)|^2 \right) dt$$

$$= \int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} rect(t/2 - 2k) \right) dt = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} rect(t/2 - 2k) dt$$

$$= \sum_{k=-\infty}^{\infty} 2$$

$$\boxed{E_x = \infty}$$

Since the signal is periodic, with period 4,

$$P_x = \frac{1}{4} \int_0^4 |x(t)|^2 dt$$

$$= \frac{1}{4} \int_0^4 \left(rect(t/2) + rect(t/2 - 2) + rect(t/2 - 4) \right) dt$$

$$= \frac{1}{4} \left(\int_0^4 rect(t/2) dt + \int_0^4 rect(t/2 - 2) dt + \int_0^4 rect(t/2 - 4) dt \right.$$

$$= \frac{1}{4} \left(1 + 2 + 1 \right) = \frac{4}{4} = 1$$

$$\boxed{P_x = 1}$$

Homework #1 Solutions

1.b.iv. $x_{rms} = \sqrt{P_x}$

$= \sqrt{1}$

$\boxed{x_{rms} = 1}$

$M_x = \max(x(t))$

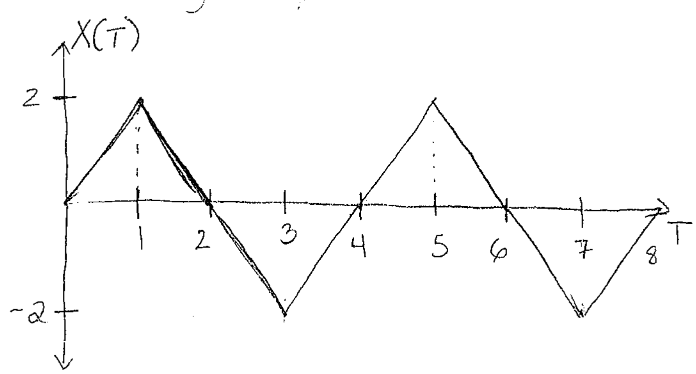
$\boxed{M_x = 1}$

$$A_x = \int_{-\infty}^{\infty} x(t)\,dt$$

Let $X(t) = \int_{-\infty}^{t} x(t)\,dt$. Then $X(t)$ oscillates as $t \to \infty$. Thus, $\boxed{A_x \text{ is undefined.}}$

$$x_{avg} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\,dt$$

Let $X(T) = \int_{-T}^{T} x(t)\,dt$. Then $X(T)$ takes the following shape.



and $X(T)$ is bounded above by 2 and bounded below by -2.

$$|x_{avg}| \le \lim_{T\to\infty} \frac{1}{2T} |X(T)| \le \lim_{T\to\infty} \frac{1}{2T}\, 2 = 0$$

$\Rightarrow \boxed{x_{avg} = 0}$

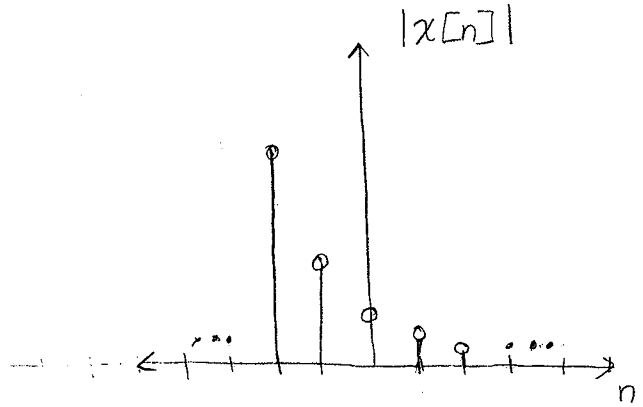
Homework #1 Solutions

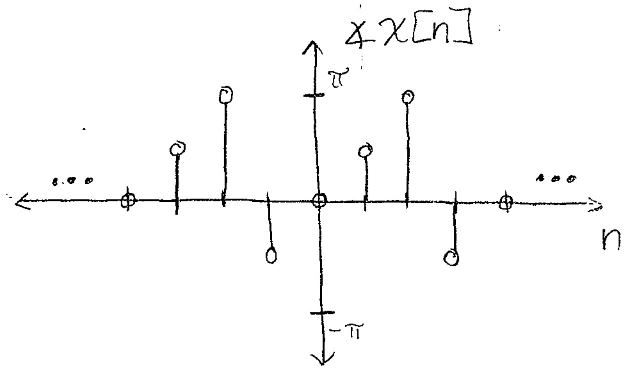
1.c. $x[n] = e^{(j-1)\pi n/2}$

$= e^{-\pi n/2} e^{j\pi n/2}$

$= (e^{-\pi/2})^n e^{j\pi n/2}$

i.





ii. two-sided

iii. neither causal nor anticausal

iv.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} |e^{(j-1)\pi n/2}|^2 = \sum_{n=-\infty}^{\infty} (e^{-\pi})^n$$

$$\boxed{E_x = \infty}$$

1.c.iv. $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{(j-1)\pi n/2}|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} e^{-\pi n}$

$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{e^{\pi N} - e^{-\pi(N+1)}}{1 - e^{-\pi}} \right)$

$\boxed{P_x = \infty}$

$x_{rms} = \sqrt{P_x}$

$= \sqrt{\infty}$

$\boxed{x_{rms} = \infty}$

$M_x = \max(x[n])$

$\boxed{M_x \text{ is does not exist}}$

$A_x = \sum_{n=-\infty}^{\infty} x[n]$

$= \sum_{n=-\infty}^{\infty} e^{(j-1)\pi n/2}$

$= \lim_{N\to\infty} \sum_{n=0}^{N} \left(e^{(j-1)\pi/2}\right)^n + \sum_{n=0}^{N} \left(e^{(1-j)\pi/2}\right)^n - 1$

$= \lim_{N\to\infty} \frac{1 - e^{[(j-1)\pi/2](N+1)}}{1 - e^{(j-1)\pi/2}} + \frac{1 - e^{[(1-j)\pi/2](N+1)}}{1 - e^{(1-j)\pi/2}} - 1$

while all other limits approach constants or zero $\lim_{N\to\infty} e^{[(1-j)\pi/2](N+1)}$ diverges

Thus, $\boxed{A_x \text{ is undefined}}$

1.c.iv. $x_{avg} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]$

from the previous calculation of A_x we know that all other limits will approach zero, so we can concentrate on the following limit.

$\lim_{N\to\infty} \frac{1}{2N+1} e^{(1-j)\pi(N+1)/2}$

This limit is also divergent. Thus,

$\boxed{x_{avg} \text{ is undefined}}$

2. (a) $-\sin(2\pi t)[u(t-2)-u(t-3)]$

(b) $\sin\left(\frac{\pi}{3}(n+1)\right)$

3. a. $y(t) = \int_{t-1}^{t} x(\tau)d\tau$

i. The system is $\boxed{\text{linear}}$

Let $y_1(t) = \int_{t-1}^{t} x_1(\tau)d\tau$ and $y_2(t) = \int_{t-1}^{t} x_2(\tau)d\tau$.

Show that $\int_{t-1}^{t} (c_1 x_1(\tau) + c_2 x_2(\tau))d\tau = c_1 y_1(t) + c_2 y_2($

$$\int_{t-1}^{t} (c_1 x_1(\tau) + c_2 x_2(\tau))d\tau = \int_{t-1}^{t} c_1 x_1(\tau)d\tau + \int_{t-1}^{t} c_1 x_2(\tau)d\tau$$

$$= c_1 \int_{t-1}^{t} x_1(\tau)d\tau + c_2 \int_{t-1}^{t} x_2(\tau)d\tau$$

$$= c_1 y_1(t) + c_2 y_2(t) \quad \text{Q.E.D.}$$

Homework #1 Solutions

3\. a. ii. The system is $\boxed{\text{time-invariant}}$

Let $y_1(t) = \int_{t-1}^{t} x(\tau)d\tau$ and $y_2(t) = \int_{t-1}^{t} x(\tau - T)d\tau$

The system is time invariant if $y_2(t) = y_1(t-T)$

Let $b = \tau - T$, then $db = d\tau$ and

$$y_2(t) = \int_{t-1}^{t} x(\tau - T)d\tau = \int_{t-1-T}^{t-T} x(b)db = \int_{(t-T)-1}^{(t-T)} x(b)db = y_1(t-T)$$

Q.E.D.

iii. The system is $\boxed{\text{causal}}$

Let $x_1(t) = x_2(t)$ for $t \le t_0$ where $t_0 \in (-\infty, \infty)$. Then $y_1(t) = \int_{t-1}^{t} x_1(\tau)d\tau$

and $y_2(t) = \int_{t-1}^{t} x_2(\tau)d\tau$ and since $x_1(t) = x_2(t)$

$y_2(t) = \int_{t-1}^{t} x_1(\tau)d\tau$ $\therefore$ $y_1(t) = y_2(t)$ whenever $t \le t_0$. So it follows by definition that the system is causal. Q.E.D.

Homework #1 Solutions

3\. a. iv. The system is $\boxed{\text{stable}}$.

Let M be a constant scalar value such that $|x(t)| < M \quad \forall \; t \in \mathbb{R}$. Then,

$$y(t) = \int_{t-1}^{t} x(\tau) d\tau \leq \int_{t-1}^{t} |x(\tau)| d\tau$$

$$\leq \int_{t-1}^{t} M \, d\tau = M(t-(t-1))$$

$\therefore \; y(t) \leq M \quad \forall \, t \in \mathbb{R}$ whenever $x(t)$ defined for $t \in \mathbb{R}$ is bounded. Q.E.D.

v. The system is $\boxed{\text{not memoryless}}$

Assume $x(t) = u(t) - u(t-5) + u(t-6)$

At time $t_1 = 0.5$, $x(t_1) = 1$ and $y(t_1) = 1$, while at time $t_2 = 6.5$ $x(t_2) = 1$ and $y(t_2) = 0.5$. Since, $y(t_1) \neq y(t_2)$ even though $x(t_1) = x(t_2)$, $y(t)$ is not memoryless. Q.E.D.

Homework #1 Solutions

3. b. $y(t) = rect(x(t))$

i. This system is $\boxed{\text{not linear.}}$

Let $x_1(t) = 0$ and $x_2(t) = 1$. $a = b = 1$

$T[x_1(t) + x_2(t)] = rect(0+1)$
$= 0$

$T[x_1(t)] + T[x_2(t)] = rect(0) + rect(1)$
$= 1 + 0 = 1$

Since $T[x_1(t) + x_2(t)] \neq T[x_1(t)] + T[x_2(t)]$

The system is not linear. Q.E.D

ii. The system is $\boxed{\text{time-invariant.}}$

Let $x_1(t)$, $x_2(t)$ be input signals and $y_1(t)$, $y_2(t)$ be the respective responses to the system, $T(\ )$.

Suppose $x_2(t) = x_1(t - t_0)$, then

$y_2(t) = rect(x_2(t))$
$= rect(x_1(t - t_0))$
$= y_1(t - t_0)$

Thus, the system is time-invariant by definition. Q.E.D.

iii. The system is $\boxed{\text{causal.}}$

Let $x_1(t) = x_2(t)$ for $t \leq t_0$, where $t_0 \in (-\infty, \infty)$. Let $y_1(t) = rect(x_1(t))$ and $y_2(t) = rect(x_2(t))$.

Since $x_1(t) = x_2(t)$ for $t \leq t_0$ then $y_2(t) = rect(x_2(t)) = rect(x_1(t)) = y_1(t)$ for $t \leq t_0$. Thus, it follows that the system is causal by definition.

Q.E.D.

3. b. iii. The system is $\boxed{\text{stable.}}$

Let $x(t) \le M_x$. Then
$y(t) = \text{rect}(x(t)) \le \text{rect}(M_x) \le 1$

Thus, it follows by definition that the system is stable.

iv. The system is $\boxed{\text{memoryless.}}$

$$y(t) = \begin{cases} 1 & \text{when } |x(t)| \le 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Since the system response is completely described by a function relating the input to the output at time t, the system is memoryless.

c. $y[n] = x[n] + (1/2)y[n-1]$

i. The system is $\boxed{\text{linear.}}$

Let $x_1[n]$; $x_2[n]$ be input signals to the system $T()$, $y_1[n], y_2[n]$ be the respective outputs from the system $T()$, and let $T()$ be defined s.t. $y[n] = T(x[n]) = \frac{1}{2} y[n-1] + x[n]$. Futhermore, let $x_3[n] = ax_1[n] + bx_2[n]$ and $y_3[n] = ay_1[n] + by_2[n]$. Then the following is true.

$$a\,y_1[n] = a(\tfrac{1}{2} y_1[n-1] + x_1[n])$$
$$+ \quad b\,y_2[n] = b(\tfrac{1}{2} y_2[n-1] + x_2[n])$$

$\Rightarrow ay_1[n] + by_2[n] = \frac{1}{2}(a\,y_1[n-1] + b\,y_2[n-1]) + ax_1[n] + bx_2[n]$

$\Rightarrow y_3[n] = \frac{1}{2} y_3[n-1] + x_3[n]$

$\Rightarrow y_3[n] = T(x_3[n])$ i.e. $ay_1[n] + by_2[n] = T(ax_1[n] + bx_2[n])$

3\. c. i. Thus, by definition, the system is linear, Q.E.D.

ii. The system is $\boxed{\text{time invariant.}}$

Let $x_2[n] = x_1[n-n_0]$ and

$y_1[n] = \frac{1}{2} y_1[n-1] + x_1[n] \Rightarrow x_1[n] = y_1[n] - \frac{1}{2} y_1[n-1]$

$y_2[n] = \frac{1}{2} y_2[n-1] + x_2[n]$.

$$y_2[n] = \frac{1}{2} y_2[n-1] + x_2[n]$$
$$y_2[n] - \frac{1}{2} y_2[n-1] = x_2[n]$$
$$y_2[n] - \frac{1}{2} y_2[n-1] = x_1[n-n_0]$$
$$y_2[n] - \frac{1}{2} y_2[n-1] = y_1[n-n_0] - \frac{1}{2} y_1[n-n_0]$$

$$\Longrightarrow y_2[n] = y_1[n-n_0]$$

Thus, the system is time invariant by definition.

iii. The system is $\boxed{\text{causal}}$

Theorem $h[n] = 0 \; \forall n < 0 \Longleftrightarrow$ the system is causal

$y[n] = \frac{1}{2} y[n-1] + x[n]$, $h[n] = ?$

$$h[n] = \frac{1}{2} h[n-1] + \delta[n]$$
$$h[-1] = 0$$
$$h[0] = 1$$
$$h[1] = \frac{1}{2}(1) = \frac{1}{2}$$
$$h[2] = \frac{1}{2}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2$$
$$h[3] = \frac{1}{2}\left(\left(\frac{1}{2}\right)^2\right) = \left(\frac{1}{2}\right)^3$$
$$\vdots$$
$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Since $h[n] = 0 \; \forall n < 0$, the system is causal according to the theorem above.

3\. c. iv. The system is $\boxed{\text{BIBO stable}}$

Theorem A LTI system is BIBO $\Longleftrightarrow \sum_{k=-\infty}^{\infty} |h[k]| < \infty$.

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |(1/2)^k u[k]|$$

$$= \sum_{k=0}^{\infty} (1/2)^k = \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

By the theorem above, the system is stable. Q.E.D.

v. The system is $\boxed{\text{not memoryless}}$

Let $x[n] = \cos\left(\frac{\pi}{2} n\right) u[n]$.

$x[0] = 1$ and $y[0] = 1$
$x[4] = 1$ and $y[4] = 19/16$

$y[1]$	$\frac{1}{2}(1) + 0 = \frac{1}{2}$
$y[2]$	$\frac{1}{2}(\frac{1}{2}) - 1 = \frac{3}{4}$
$y[3]$	$\frac{1}{2}(\frac{3}{4}) = \frac{3}{8}$
$y[4]$	$\frac{1}{2}(\frac{3}{8}) + 1 = \frac{19}{16}$

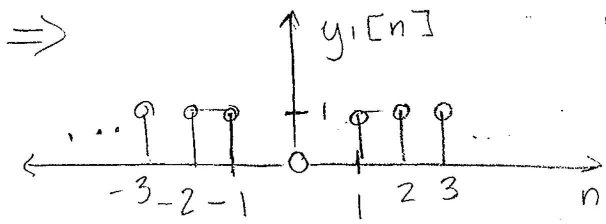
Since $y[0] \neq y[4]$ when $x[0] = x[4] = 1$. The system is not memoryless.

3.d. $y[n] = \begin{cases} x[n] & , x[n] < |n| \\ |n| & , \text{else} \end{cases}$

i. The system is not linear.

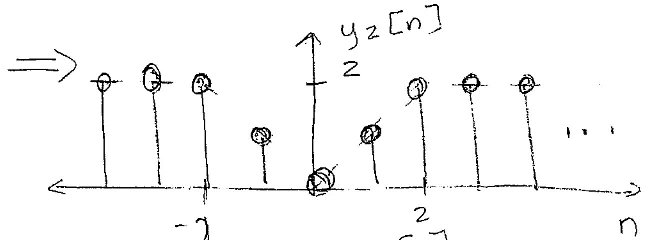
For example, let $x_1[n] = 1$, $x_2[n] = 2$ and $x_3[n] = 3$.

$\Rightarrow$



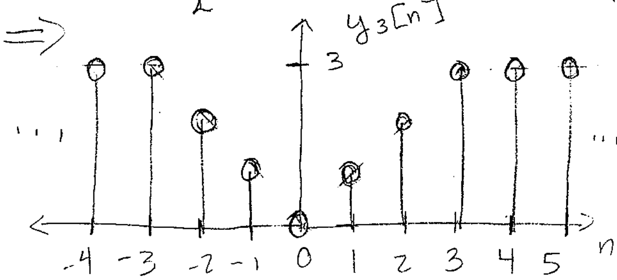
$y_1[n] = \begin{cases} |n| & , |n| < 1 \\ 1 & \text{otherwise} \end{cases}$

$\Rightarrow$



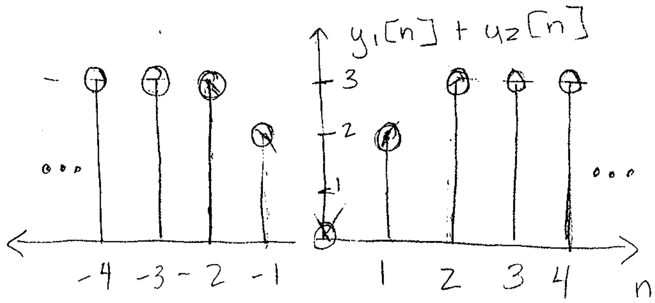
$y_2[n] = \begin{cases} |n| & , |n| < 2 \\ 2 & \text{otherwise} \end{cases}$

$\Rightarrow$



$y_3[n] = \begin{cases} |n|, & |n| < 3 \\ 3 & \text{otherwise} \end{cases}$

But $x_3[n] = x_1[n] + x_2[n]$. If the system is linear, then $y_3[n] = y_1[n] + y_2[n]$



$y_1[n] + y_2[n] = \begin{cases} 2|n| & \text{for } |n| < 1 \\ |n| + 1 & \text{for } 1 < |n-1| < 2 \\ 3 & \text{otherwise} \end{cases}$

3.d.i. $\Rightarrow y_3[n] \neq y_1[n] + y_2[n]$

Thus, the system is not linear. QED

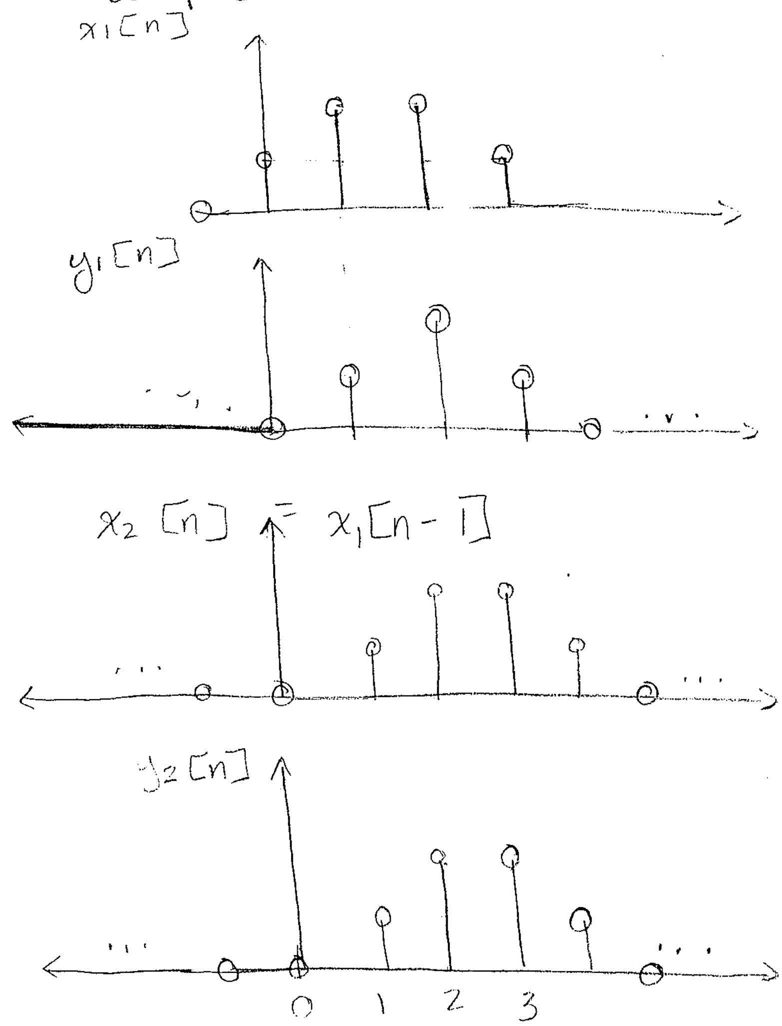
ii. The system is not time invariant

Let $x_2[n] = x_1[n-n_0]$

Then $y_2[n] = \begin{cases} x_2[n] & \text{for } x_2[n] < |n| \\ |n| & \text{otherwise} \end{cases}$

$y_2[n] = \begin{cases} x_1[n-n_0] & \text{for } x_1[n-n_0] < |n| \\ |n| & \text{otherwise} \end{cases}$

Example



$y_1[n-1] \neq y_2[n]$

$\therefore$ The system is not time-invariant

3, d. iii The system is $\boxed{\text{causal.}}$

Let $x_1[n] = x_2[n]$ for $n \le n_0$, where $n_0 \in (-\infty, \infty)$. Let

$$y_1[n] = \begin{cases} x_1[n] & \text{for } x_1[n] < |n| \\ |n| & \text{otherwise} \end{cases}$$

and

$$y_2[n] = \begin{cases} x_2[n] & \text{for } x_2[n] < |n| \\ |n| & \text{otherwise} \end{cases}$$

Then since $x_1[n] = x_2[n]$ for $n \le n_0$

$$y_2[n] = \begin{cases} x_1[n] & \text{for } x_1[n] < |n| \\ |n| & \text{otherwise} \end{cases} \quad \text{for } n \le n_0$$

$$= y_1[n] \quad \text{for } n \le n_0$$

Thus, the system is causal. Q.E.D.

iv. The system is BIBO stable

Let $|x[n]| \le M_x < \infty$, and.

$$y[n] = \begin{cases} x[n] & \text{for } x_1[n] < |n| \\ |n| & \text{otherwise} \end{cases}$$

Since $|x[n]| \le M_x$, for

$$|y[n]| = \begin{cases} |x[n]| & \text{for } n > M_x \\ |x[n]| & \text{for } n < M_x \ \& \ x[n] < |n| \\ |n| & \text{for } n < M_x \ \& \ x[n] \ge |n| \end{cases}$$

$$\Rightarrow |y[n]| \le M_x < \infty$$

$\therefore$ The system is BIBO stable by definition. Q.E.D.

3.d.v. The system is not memoryless.

Let $x_1 = x[n_1]$ and $x_2 = x[n_2]$
and $y_1 = T(x(n_1))$ and $y_2 = T(x(n_2))$
where $y = T(x(n))$

$$= \begin{cases} x(n) & x(n) < |n| \\ |n| & \text{otherwise} \end{cases}$$

Let $n_1 = 1$ and $n_2 = 5$
and $x[n_1] = x[n_2] = 2$.
Then $y_1 = 1$ and $y_2 = 2$.
Since $y_1 \neq y_2$, the system $T(\ )$
is not memoryless.

4. a. $x[n] = u[n] - u[n-4]$; $h[n] = a^n u[n]$

$$y[n] = h[n] * x[n]$$
$$= h[n] * (u[n] - u[n-4])$$
$$= h[n] * u[n] - h[n] * u[n-4]$$

$$h[n] * u[n] = \sum_{k=-\infty}^{\infty} a^k u[k]\, u[n-k]$$
$$= \left(\sum_{k=0}^{n} a^k\right) u[n] = \frac{1-a^{n+1}}{1-a} u[n]$$

Since the system is LTI,

$$h[n] * u[n-4] = \frac{1-a^{(n-4)+1}}{1-a} u[n-4]$$
$$= \frac{1-a^{n-3}}{1-a} u[n-4]$$

$$y[n] = \left(\frac{1-a^{n+1}}{1-a}\right) u[n] - \left(\frac{1-a^{n-3}}{1-a}\right) u[n-4]$$

4. b. $x[n] = a^n u[n]$, $h[n] = b^n u[n]$ $(a \neq b)$
$(a, b < 1)$

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$
$$= \sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$$
$$= \left(b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k \right) u[n]$$

$$\boxed{y[n] = b^n \left(\frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \left(\frac{a}{b}\right)} \right) u[n]}$$

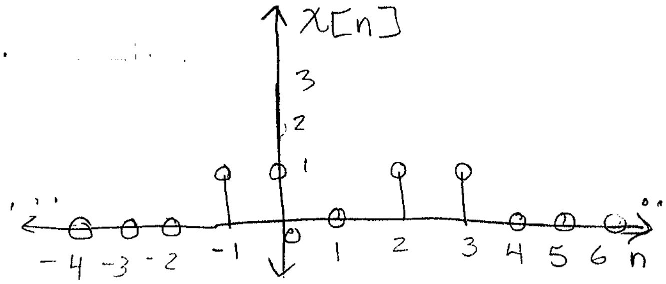
c. $x[n] = a^n u[n]$, $h[n] = a^n u[n]$

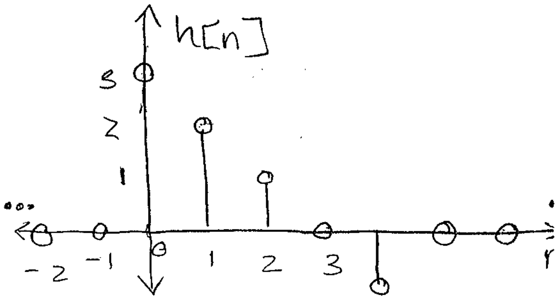
$$y[n] = x[n] * h[n]$$

$$y[n] = \left(\lim_{b \to a} b^n \left(\frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \left(\frac{a}{b}\right)} \right) \right) u[n]$$
$$= \left(\lim_{b \to a} \left(\frac{b^n - \frac{a^{n+1}}{b}}{\frac{b-a}{b}} \right) \right) u[n]$$
$$= \left(\lim_{b \to a} \left(\frac{b^{n+1} - a^{n+1}}{b-a} \right) \right) u[n]$$
$$= \left(\lim_{b \to a} \left(\frac{(n+1) b^n}{1} \right) \right) u[n]$$

$$\boxed{y[n] = (n+1) a^n u[n]}$$

4. a.





$x[n] = \{1, \underset{\uparrow}{1}, 0, 1, 1\}$; $h[n] = \{\underset{\uparrow}{3}, 2, 1, 0, -1\}$

$y[n] = x[n] * h[n]$

Use the sliding tape method

1	1	0	1	1				
				3	2	1	0	-1

← flip $x[n]$ and shift one to the left to get $y[-1]$

$y[-1] = 1 \times 3 = 3$

1	1	0	1	1			
			3	2	1	0	-1

$y[0] = 1 \times 3 + 1 \times 2 = 5$

1	1	0	1	1		
		3	2	1	0	-1

$y[1] = 0 \times 3 + 1 \times 2 + 1 \times 1 = 3$

1	1	0	1	1	
	3	2	1	0	-1

$y[2] = 1 \times 3 + 0 \times 2 + 1 \times 1 + 1 \times 0 = 4$

1	1	0	1	1
3	2	1	0	-1

$y[3] = 1 \times 3 + 1 \times 2 + 0 \times 1 + 1 \times 0 + 1 \times -1 = 4$

4.d. Similarly,

$$y[4] = 1\times 2 + 1\times 1 + 0\times 0 + 1\times -1 = 2$$
$$y[5] = 1\times 1 + 1\times 0 + 0\times -1 = 1$$
$$y[6] = 1\times 0 + 1\times -1 = -1$$
$$y[7] = 1\times -1 = -1$$

$$\therefore y[n] = \{3, \underset{\uparrow}{5}, 3, 4, 4, 2, 1, -1, -1\}$$

The length of $y[n]$ is 9 and is equal to the sum of the length of $x[n]$, 5, and $h[n]$, 5, minus 1.

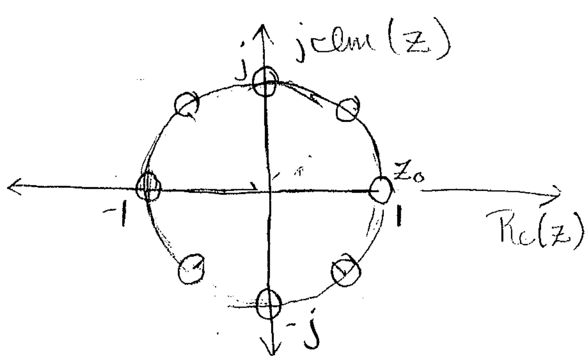
i.e. $5 + 5 - 1 = 9$.

Note: It is good practice to check this whenever using this method, just to make sure you haven't missed any important value.

5. $z^N - w = 0$, find z_i, $i \in \{0, \dots, N-1\}$

a. $N=8$, $w=1$

$$z^8 = 1 = e^{j((0) + 2\pi i)}$$
$$z_i = (e^{j(2\pi i)})^{1/8}$$
$$z_i = e^{j(\frac{2\pi}{8} i)}$$
$$\boxed{z_i = e^{j(\frac{\pi}{4} i)} \quad i \in \{0, 1, \dots, 7\}}$$



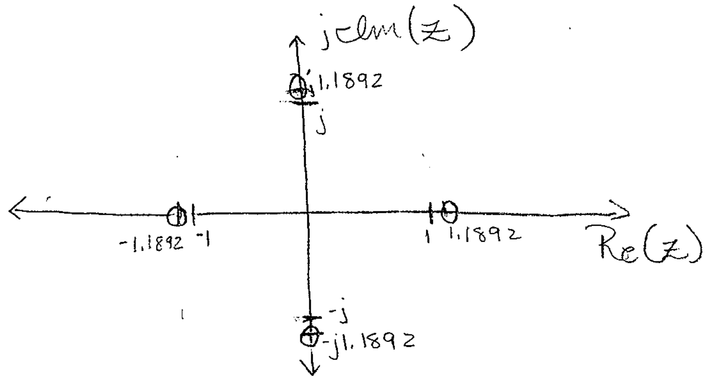
5. b. $z^N - w = 0$

$z^4 - 2e^{j(0)} = 0$

$z_i = \left(2e^{j(2\pi i)}\right)^{1/4}$

$$\boxed{z_i = \sqrt[4]{2}\; e^{j(\frac{\pi}{2} i)} \quad \text{for } i \in \{0,1,2,3\}}$$

$\sqrt[4]{2} \approx 1.1892$



c. $N=5$, $w=j$

$z_i = \left(e^{j(\frac{\pi}{2} + 2\pi i)}\right)^{1/5}$

$$\boxed{z_i = e^{j(\frac{\pi}{2} + \frac{2\pi}{5} i)} \quad i \in \{0,1,2,3,4\}}$$