EE 438 Digital Signal Processing with Applications
Homework #8 due 11/28/2007

1. Find expressions for the N point DFT's of the following signals.

   a. $x(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n = 1, \ldots, N-1 \end{cases}$

   b. $x(n) = (-1)^n$, $n = 0, \ldots, N-1$, (Assume that N is even)

   c. $x(n) = e^{j2\pi nk/N}$, $n = 0, \ldots, N-1$ where k is an integer between 0 and $N-1$.

   d. $x(n) = \cos(2\pi nk/N)$, $n = 0, \ldots, N-1$ where k is an integer between 0 and $N-1$.

2. The objective of this problem is to show that the DFT and inverse DFT formulas are correct. In other words, the object is to verify that when the DFT is computed via

$$X_N(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}},$$

   then the inverse DFT computed via

$$z(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_N(k)\, e^{j\frac{2\pi nk}{N}},$$

   produces a function $z(n)$ so that $z(n) = x(n)$. To do this, we first must show that

$$\sum_{k=0}^{N-1} e^{j2\pi kn/N} = N \sum_{m=-\infty}^{\infty} \delta(n - mN).$$

   Define the function $y(n) = \sum_{k=0}^{N-1} e^{j2\pi kn/N}$ and show the following:

   a) Show that $y(n)$ is periodic with period N.
   b) Show that $y(0) = N$.
   c) Show that $y(n) = 0$ for $n = 1, \ldots, N-1$.
   d) Next plug in the expression $X_N(k) = \sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi mk}{N}}$ into the expression for the inverse DFT to form the expression $z(n) = \frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi mk}{N}}\right\} e^{j\frac{2\pi nk}{N}}$. Then simplify the expression for $z(n)$ using the result above, and show that $z(n) = x(n)$.

3. Compute the DFT of the following signals

   a) $w_L(n) = \begin{cases} 1 & \text{for } 0 \le n < L \\ 0 & \text{otherwise} \end{cases}$ for $L \le N$

   b) $x(n) = e^{j\omega_0 n} w_L(n)$ for $L \le N$ and $0 < \omega_0 < \pi$