EE 438 Digital Signal Processing with Applications
Homework #7 due 11/12/2007

1. Consider a discrete-time LTI system with impulse response $h(n)$. Prove that the system is BIBO stable if and only if $h(n)$ is absolutely summable.

2. Let the impulse response $h(n)$ have a Z-transform $H(z)$ with a region of convergence of Ω. Prove that $h(n)$ is absolutely summable if and only if $1 \in \Omega$.

3. Let the impulse response $h(n)$ have a rational Z-transform $H(z)$ with a region of convergence of Ω.
   - i) If $\{z : |z| > a\} \subset \Omega$ for $0 < a < \infty$, than what property does $h(n)$ have?
   - ii) If $\{z : |z| < a\} \subset \Omega$ for $0 < a < \infty$, than what property does $h(n)$ have?
   - iii) If $0 \in \Omega$, than what property does $h(n)$ have?
   - iv) If $\infty \in \Omega$, than what property does $h(n)$ have?

4. Consider the ZT

$$H(z) = \frac{z^2 - z}{(z^2 - 1/4)(z - 1/2)}.$$

   - a) Find the poles and zeros of this ZT.
   - b) Sketch the poles, zeros and the possible ROC's.
   - c) For each ROC, determine if the impulse response is causal, right sided, or left sided.
   - d) For each ROC, determine if the impulse response is stable or unstable.
   - e) For each ROC, compute the corresponding signal $h(n)$.
   - f) Find a difference equation which implements this transfer function, and draw its flow diagram.

5. Consider the following difference equation

$$y(n) = ay(n-1) + x(n) - x(n-1).$$

   - a) Compute the transfer function $H(z) = \dfrac{Y(z)}{X(z)}$, and find its poles and zeros.
   - b) Compute the impulse response $h(n)$ using a ROC of $|z| > a$. For what values of a is the system stable?
   - c) Compute the impulse response $h(n)$ using a ROC of $|z| < a$. For what values of a is the system stable?

6. Consider the causal D-T LTI system described by the following recursive difference equation

$$y(n) = x(n) - x(n-8) + y(n-1)$$

a) Find the transfer function $H(z)$ for this filter.
b) Sketch the locations of poles and zeros in the complex z-plane.
c) For each ROC, find the impulse response $h(n)$ by computing the inverse ZT of $H(z)$.
d) Is this filter IIR or FIR? Explain your answer.