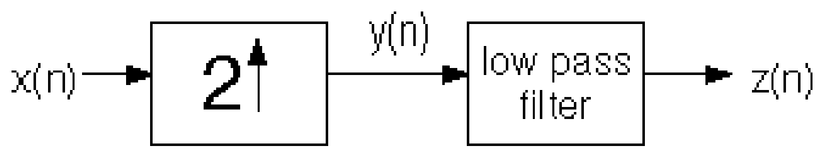
EE 438 Digital Signal Processing with Applications
Homework #6 due 10/19/2007

1. Spend 2 hours studying the notes, and making sure you fully understand the concepts of:
   a) Sampling
   b) Recontruction
   c) Zero order hold (i.e. the model of a physical D/A converter)
   Do this in a quite room with no disturbances.

2. Consider a digital signal processing system with the following sequence of operations:
   a. The CT signal $x(t)$ is sampled at period T to form the DT signal $y(n)$
   b. The DT signal $y(n)$ is filtered by the filter $H\left(e^{j\omega}\right)$ to form the signal DT $z(n)$.
   c. The DT signal $z(n)$ is converted to a continuous time signal $r(t)$ using a zero order hold.
   d. The CT signal $r(t)$ is filtered with a perfect LPF with a cutoff of $1/(2T)$ to form the CT signal $x_r(t)$.

   Derive expressions for the frequency transforms of each signal. Assuming that the signal $x(t)$ is bandlimited to frequency $1/(2T)$, calculate the frequency response for $H\left(e^{j\omega}\right)$ that is required to achieve perfect reconstruction of $x(t)$.

3. The following system shows an interpolator with discrete input $x(n)$. Assume that the low pass filter has frequency response $H(e^{j\omega}) = 2\text{rect}(\omega/\pi)$ for $|\omega| < \pi$.



   Compute the output $z(n)$ for the following inputs.
   a. $x(n) = \delta(n)$
   b. $x(n) = \delta(n-1)$
   c. $x(n) = 1$
   d. $x(n) = \cos(\pi n / 4)$
   e. $x(n) = \text{sinc}(n / 8)$

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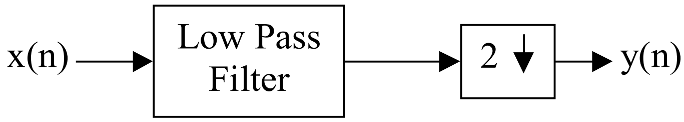

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c) $x(n) = 1$

d) $x(n) = \cos(\pi n / 4)$

e) $x(n) = \mathrm{sinc}(n/8)$

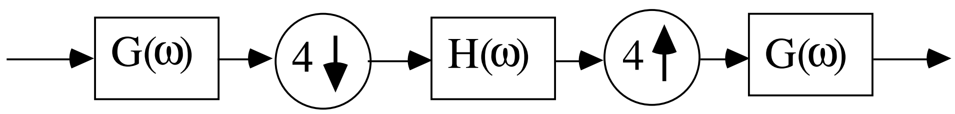
5. Consider the digital filter described by the following difference equation

$$y(n) = 0.25\,\big(x(n+1) + 2\,x(n) + x(n-1)\big)$$

a) Find a simple expression for the frequency response $H(e^{j\omega})$ of this filter.

b) Sketch the magnitude and phase of $H(e^{j\omega})$.

Now consider the following digital system,



where $H(e^{j\omega})$ is the filter from parts a and b and $G(e^{j\omega})$ is an ideal low-pass filter with a cutoff frequency of $\pi/4$ rad/sample and unity gain in the passband.

c) Find the overall frequency response $F(e^{j\omega})$ for this system

d) Sketch the magnitude and phase of $F(e^{j\omega})$.

e) Discuss the possible advantages of a system like that shown above compared to directly implementing a digital filter with frequency response $F(e^{j\omega})$ as a single stage.