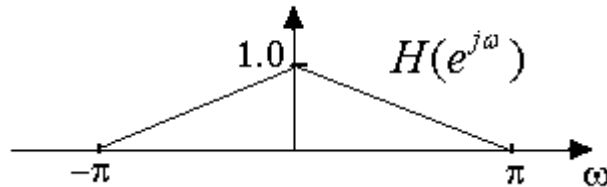


**EE 438 Digital Signal Processing with Applications**  
**Homework #5 due 10/10/2007**

1. Let  $y(n)$  be a D-T signal formed by sampling the C-T signal  $x(t)$  so that  $y(n) = x(nT)$ . For each case do the following:
  - i) calculate an analytical expression for  $Y(e^{j\omega})$
  - ii) use Matlab to plot  $y(n)$
  - iii) use Matlab to plot the magnitude and phase of  $Y(e^{j\omega})$  for  $\omega \in [-\pi, \pi]$ .
  - a)  $x(t) = \text{sinc}(t), T = 1$
  - b)  $x(t) = \text{sinc}(t - 1/4), T = 1$
  - c)  $x(t) = \text{sinc}(t), T = 1/2$
  - d)  $x(t) = \text{sinc}(t), T = 3/2$

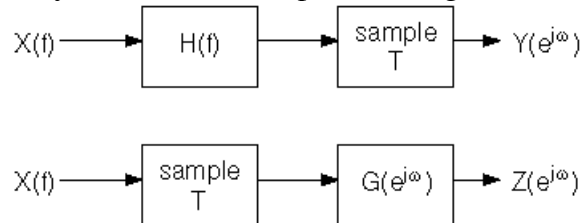
2. The analog signal  $x(t) = \sin(2\pi f_0 t)$  is sampled with an ideal sampler at a rate of 8 kHz, filtered with a digital filter having the frequency response  $H(e^{j\omega})$  shown below, and then reconstructed as an analog signal  $y(t)$  with an impulse generator followed by an ideal low pass filter with a cutoff frequency of 4 kHz.



Find the output  $y(t)$  for the following values of  $f_0$ .

- a. 1 kHz,
- b. 5 kHz

3. The following two systems are used to process a signal



Assume that the input signal  $X(f)$  and the continuous time filter  $H(f)$  are both band limited to  $1/(2T)$ .

- a) Compute analytic expressions for  $Y(e^{j\omega})$  and  $Z(e^{j\omega})$ .
- b) Show that if

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

then the outputs of the two systems are equal.

- c) Explain what (if any) advantages the second system may have.
4. You decide that you must have a programmable time delay for your new band. In order to make the design flexible you decide to sample the signal, process it digitally, and then reconstruct it. You band limit the signal to  $20\text{kHz}$ , use a sampling rate of  $44\text{kHz}$ , and reconstruct with an impulse generator followed by an ideal low pass filter with a cut-off frequency of  $22\text{kHz}$ . Assuming that there is no delay in the sampling and reconstruction process:
- Find the digital filter  $H(e^{j\omega})$  that will produce a pure time delay of  $d$  sec.  
(Hint: Find a digital filter so that the ratio  $X_r(f)/X(f) = e^{-j2\pi fd}$ .)
  - Plot the phase and magnitude of  $H(e^{j\omega})$  over the interval  $[0, 2\pi]$  for i)  $d = 22.727 \mu\text{sec} = 1/44 \text{ kHz}$  ii)  $d = 35 \mu\text{sec}$ .
  - What is the impulse response  $h(n)$  when  $d = 22.727 \mu\text{sec}$ .