EE 438 Digital Signal Processing with Applications
Homework #4 due 9/24/2007

1) Calculate the DTFT for each of the following signals where $|a| < 1$

   a) $x(n) = \int_{-\pi}^{\pi} \left(\frac{\cos(\omega)^3}{1 + |\omega|^{3/2}} \right) e^{j\omega n} d\omega$

   b) $x(n) = a^n u(n)$

   c) $x(n) = a^n e^{j\omega_0 n} u(n)$

   d) $x(n) = u(n) - u(n - N)$

2) Prove that for a LTI system with input $x(n)$, and output $y(n)$, the output is given by $y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$ where $h(n)$ is the output when $x(n) = \delta(n)$ i.e. $h(n)$ is the impulse response.

3) Consider a DT LTI system with impulse response $h(n)$, input $x(n)$, and output $y(n)$. Show that when the input is $x(n) = e^{j\omega_0 n}$, then the form of the output is $y(n) = Ce^{j\omega_0 n}$, and determine an expression for the complex number C.

4) Consider a DT LTI system with transfer function $H(e^{j\omega})$, input $x(n)$, and output $y(n)$. Calculate the output for the following inputs.

   a) $x(n) = e^{j\omega_0 n}$

   b) $x(n) = e^{j(\omega_0 n + \phi)}$

   c) $x(n) = \cos(\omega_0 n)$

   d) $x(n) = \cos(\omega_0 n + \phi)$

   e) $x(n) = \sin(\omega_0 n)$

   f) $x(n) = \sin(\omega_0 n + \phi)$

5) Consider a DT LTI system described by the following equation $y(n) = x(n) + 0.5y(n-1)$.

   a) Compute the impulse response $h(n)$ of the system.

   b) Compute the frequency response $H(e^{j\omega})$ of the system by taking the DTFT of the difference equation.

   c) Compute the DTFT of $h(n)$ and verify that it is the same as $H(e^{j\omega})$.

6) For the LTI systems below,

   i. find the impulse response,

   ii. find an expression for the frequency response (simplify as much as possible),

   iii. sketch the magnitude and phase of the frequency response,

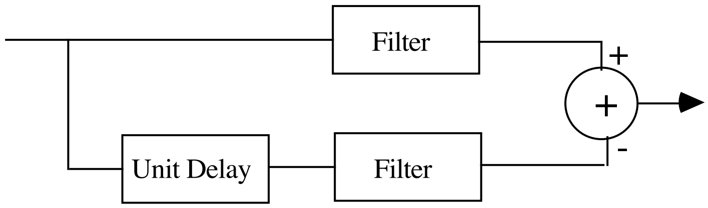
   iv. describe in general terms the effect that the filter has on a signal.

   a. $y[n] = (x[n] + x[n-1])/2$

   b. $y[n] = (x[n] + x[n-2])/2$

   c. $y[n] = x[n] + x[n-1] + y[n-1]$

7) Consider the system shown where both filters have impulse response $h_0(n)$ and frequency response $H_0(e^{j\omega})$.



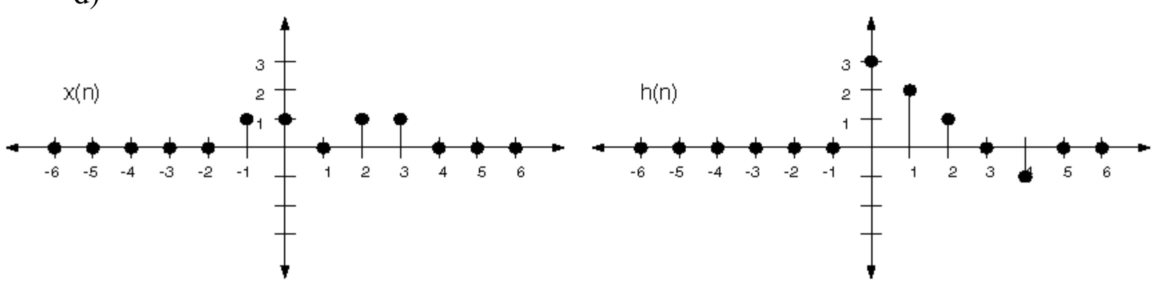
a) Determine an expression for the impulse response $h(n)$ of the overall system in terms of the impulse response of the filter $h_0(n)$.
b) Determine an expression for the frequency response $H(e^{j\omega})$ of the overall system in terms of the impulse response of the filter $H_0(e^{j\omega})$.

For the following parts, assume that the filters obey the difference equation $y[n] = (x[n] + x[n-1])/2$

c) Calculate $h_0(n)$, the impulse response of each filter.
d) Calculate $H_0(e^{j\omega})$, the frequency response of each filter.
e) Calculate the impulse response of the overall system $h(n)$.
f) Calculate the frequency response of the overall system.

8) An LTI system has input $x(n)$ and impulse response $h(n)$. Compute the output $y(n)$ for each of the following cases ($a, b < 1$).

a) $x(n) = u(n) - u(n-4)$; $h(n) = a^n u(n)$
b) $x(n) = a^n u(n)$; $h(n) = b^n u(n)$ $(a \neq b)$
c) $x(n) = a^n u(n)$; $h(n) = a^n u(n)$
d)



9) A system has the following input/output behavior.

$$y(n) = \sum_{k=-\infty}^{n} (1/2)^{n-k} x(k)$$

For each of the following parts **justify** your answer and be specific.

a) Show this system is linear.
b) Show this system is time invariant.
c) Compute the impulse response $h(n)$.
d) Is the system stable?

e) Compute $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$.

f) Compute $y(n)$ when $x(n) = \cos(\pi n)$.

g) Write a difference equation for this system