EE301 Homework #9

Problem 1 *Deriving CTFT Properties*

Derive each of the following CTFT properties. Assume that in each case the CTFT of $x(t)$ and $y(t)$ are $X(\omega)$ and $Y(\omega)$ respectively.

a) $\alpha x(t) + \beta y(t) \overset{CTFT}{\Leftrightarrow} \alpha X(\omega) + \beta Y(\omega)$

b) $x(-t) \overset{CTFT}{\Leftrightarrow} X(-\omega)$

c) $x(t - t_0) \overset{CTFT}{\Leftrightarrow} X(\omega)e^{j\omega t_0}$

d) $x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|}X(\frac{\omega}{a})$

e) $X(\omega) = X^*(-\omega)$ if $x(t)$ is real

f) $X(t) \overset{CTFT}{\Leftrightarrow} 2\pi x(-\omega)$

g) $x(t)e^{j\omega_0 t} \overset{CTFT}{\Leftrightarrow} X(\omega - \omega_0)$

h) $x(t)y(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{2\pi}X(\omega) * Y(\omega)$

i) $x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(\omega)Y(\omega)$

j) $\frac{dx(t)}{dt} \overset{CTFT}{\Leftrightarrow} j\omega X(\omega)$

Problem 2 *Computing CTFT Transforms*

For each of the following functions, compute the CTFT then sketch the function $x(t)$ and its Fourier transform $X(\omega)$.

a)
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k/2)$$

b)
$$x(t) = \text{sinc}(t) \sum_{k=-\infty}^{\infty} \delta(t - k/2)$$

c)
$$x(t) = \text{sinc}(t) \sum_{k=-\infty}^{\infty} \delta(t - k)$$

Problem 3 *Frequency analysis of linear differential equations*

Consider the system with input $x(t)$ and output $y(t)$ described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} + x(t)$$

where the system is assumed to be initially at rest.

a) Prove that the system is linear.

b) Prove that the system is time invariant.

c) Calculate the frequency response of the system $H(\omega)$.

d) Express $H(\omega)$ as the ratio of factored polynomials.