EE301 Homework #7

Problem 1 *Parseval's formula.*
Consider a signal $x(t)$ with period T with the Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \tag{1}$$

a) Derive Parseval's formula

$$\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \tag{2}$$

b) Let $x_n(t)$ be defined as

$$x_n(t) = \sum_{k=-n}^{n} a_k e^{jk\omega_0 t} \ ,$$

and let the approximation error be defined as

$$\epsilon_n(t) = x(t) - x_n(t) \ .$$

Use Parseval's formula to derive an expression for the error power

$$P_n = \frac{1}{T}\int_{-T/2}^{T/2} |\epsilon_n(t)|^2\, dt$$

c) Show that $\lim_{n\to\infty} P_n = 0$.

Problem 2 *LTI systems.*
Suppose the signal $x(t) = \mathrm{rect}(t)$ with period $T = 2$ is the input to an LTI system with impulse response $h(t) = e^{-2t}u(t)$.

(a) Compute the Fourier series coefficients, a_k, of the input, $x(t)$.

(b) Compute the Fourier series coefficients, b_k, of the output, $y(t)$.

(c) Use Parseval's theorem to compute the power of the input signal $x(t)$.

(d) Use Parseval's theorem to compute the power of the output signal $y(t)$.