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EE301 Homework #4

Problem 1 *Properties of logic.*

Prove the following basic logical relationships by using a truth table. Please refer to the notes for the structure of a truth table and the definitions of the symbols &, $\vee$, $\neg$, $\Rightarrow$, and $\Leftrightarrow$.

(a) Show that $P \Rightarrow Q$ is equivalent to $\neg Q \Rightarrow \neg P$.

(b) Show that $\neg(P \vee Q)$ is equivalent to $\neg Q \& \neg P$.

(c) Show that $\neg(P \Rightarrow Q)$ is equivalent to $P \& \neg Q$.

(d) Show that $\neg(P \& Q)$ is equivalent to $\neg P \vee \neg Q$.

Problem 2 *Properties of convolution.*

(a) Consider a CT LTI system $y(t) = x(t) * h(t)$. Show the input $\frac{dx(t)}{d(t)}$ results in the output $\frac{dy(t)}{d(t)}$.

(b) Consider the DT LTI system $y[n] = x[n] * h[n]$. Prove that

$$\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n] \right) \left(\sum_{n=-\infty}^{\infty} h[n] \right)$$

(c) Consider a CT LTI system $y(t) = x(t) * h(t)$. Prove that if $x(t)$ is periodic with period T, then $y(t)$ is also periodic with period T.

Problem 3 *Properties of convolution.*

Let $x[n]$ be a signal which is nonzero only in the interval $0 \leq n < M$ and $h[n]$ be a signal which is nonzero only in the interval $0 \leq n < N$.

(a) Determine the interval $L_1 \leq n \leq L_2$ over which $y[n] = x[n] * h[n]$ is nonzero. Express L_1 and L_2 in terms of M, and N.

(b) Verify the result in the the previous part by analytically computing the convolution of the signals $x[n] = 2(u[n] - u[n-6])$ and $h[n] = 3(u[n] - u[n-2])$.

(c) Verify the result in the the previous part by graphically computing the convolution of the signals $x[n] = 2(u[n] - u[n-6])$ and $h[n] = 3(u[n] - u[n-2])$.

Problem 4 *DT Impulse Response*

Consider the DT LTI system described by the equation

$$y[n] = x[n] - 2x[n-1] + x[n-2]$$

(a) Compute the impulse response of the system.

(b) Express the system in the form $y[n] = x[n] * h[n]$.

(c) Find the output when the input is given by $x[n] = u[n]$.

(d) Find the output when the input is given by $x[n] = 1$.

Problem 5 *DT Impulse Response*

Consider the DT LTI system described by the equation

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$

where $\lim_{n\to-\infty} y[n] = 0$.

(a) Compute the impulse response of the system.

(b) Express the system in the form $y[n] = x[n] * h[n]$.

(c) Find the output when the input is given by $x[n] = u[n]$.

(d) Find the output when the input is given by $x[n] = 1$.