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EE301 Homework #3

Problem 1 *Determining the impulse response.*

For each of the following problems, prove that the system is time invariant, or time varying. If the system is time invariant, compute its impulse response. Simplify your answer as much as possible.

(a) $y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1])$

(b) $y[n] = \sum_{k=0}^{n} x[k]$

(c) $y(t) = \int_{-\infty}^{\infty} \frac{sin(t-\tau)}{2(t-\tau)} x(t-\tau)\, d\tau$

Problem 2 *Properties of LTI systems.*

A time invariant system $T[\cdot]$ is observed to have the following input/output relationships.

$$\begin{aligned} \delta[n-1] + 2\delta[n-2] &= T\left[\, \delta[n] + 2\delta[n-2] \,\right] \\ \delta[n-1] + 2\delta[n-3] &= T\left[\, 3\delta[n-2] \,\right] \\ \delta[n+1] + 2\delta[n] + \delta[n-1] &= T\left[\, \delta[n-3] \,\right] \end{aligned}$$

(a) Prove that the system is linear or nonlinear.

(b) Compute the response to an input of $\delta[n]$, that is compute $T\left[\delta[n]\right]$.

Problem 3 *Properties of LTI systems.*

Prove the following properties.

(a) The commutative property of convolution, that is, $x(t) * y(t) = y(t) * x(t)$.

(b) The associative property of DT convolution, that is,

$$(x[n] * y[n]) * z[n] = y[n] * (x[n] * z[n]) \ .$$

(c) A DT system is causal if and only if we have $h[n] = 0$ for $n < 0$.

Problem 4 *Doing convolutions.*

(a) The impulse response of a CT LTI system is given by $h(t) = u(t+1) - u(t-1)$.

  (i) Find the output, $y_1(t)$, of the system corresponding to the input $x_1(t) = u(t) - u(t-2)$.

  (ii) Find the output, $y_2(t)$, of the system corresponding to the input $x_2(t) = u(t-1) + u(t-3) - 2u(t-5)$. (Hint: Try to write $x_2(t)$ in terms of $x_1(t)$.)

(b) Problem 2.21 parts (a), (c), and (d) from the textbook.

(c) Problem 2.22 parts (c) and (e) from the textbook.