

## EE301 Homework #2

### Problem 1 *Properties of the CT impulse function.*

Simplify the following expressions where  $x(t)$  is a continuous function of  $t$ .

(a)  $\int_{-\infty}^{\infty} x(t) \delta(t) dt$

(b)  $\int_{-\infty}^{\infty} x(t) \delta(t - 3) dt$

(c)  $\int_{-\infty}^{\infty} x(t) \delta(2t) dt$  (Hint: Apply the substitution of variables  $\tau = 2t$ .)

### Problem 2 *Properties of the DT impulse function.*

Simplify the following expressions where  $x[n]$  is a discrete-time function.

(a)  $\sum_{n=-\infty}^{\infty} x[n] \delta[n]$

(b)  $\sum_{n=-\infty}^{\infty} x[n] \delta[n - k]$

(c)  $\sum_{n=-\infty}^{\infty} x[n] x[k] \delta[n - k]$

### Problem 3 *Working with integrals and sums.*

Evaluate the following expressions.

(a)  $\sum_{k=0}^{\infty} a^k$  where  $|a| < 1$ .

(b)  $\sum_{k=0}^{N-1} a^k$

(c)  $\sum_{k=0}^{N-1} e^{j2\pi k/N}$

(d)  $\int_{-\infty}^{\infty} (t - \tau)u(t - \tau)u(\tau) d\tau$

### Problem 4

Consider a CT system with input  $x(t)$  and output  $y(t)$ . For each of the following systems, i) prove that it is linear or give a counter example. ii) prove that it is time-invariant or give a counter example.

(a)  $y(t) = \cos(t)x(t)$

(b)  $y(t) = x(\cos(t))$

(c)  $y(t) = \cos(x(t))$

(d)  $y(t) = \frac{dx(t)}{dt}$

(e)  $y(t) = \int_0^t x(\tau) d\tau$

**Problem 5**

Consider a CT system with input  $x(t)$  and output  $y(t)$ . For each of the following systems, i) determine whether it is causal or noncausal, ii) determine if it is a memoryless or memory system.

(a)  $y(t) = x(t)$

(b)  $y(t) = x(t + 1)$

(c)  $y(t) = x(t - 1)$

(d)  $y(t) = x(0)$

(e)  $y(t) = \int_0^t x(\tau) d\tau$

**Problem 6 DT System Properties.**

Consider a system with input  $x[n]$  and output  $y[n]$ . For each of the following systems, i) prove that it is linear or give a counter example, ii) prove that it is time-invariant or give a counter example.

(a)  $y[n] = x[n] + 2$

(b)  $y[n] = x[2n]$  (This operation is known as *decimation*.)

(c)  $y[n] = x[n - 1] + x[n]$

(d)  $y[n] = (x[n - 1])(x[n])$

**Problem 7 MATLAB plot.**

On a single graph (use the *hold* command), plot the real and imaginary components of the DT complex exponential function

$$x[n] = e^{-j2\pi n/3}$$

on the interval  $-20 \leq n \leq 20$ . Use the MATLAB *stem* command to do the plots, and use the *xlabel*, *ylabel*, *title*, and *legend* commands to fully annotate your plot. Turn in your plot together with a print out of your MATLAB script. Make sure all axes are labeled and the graphs are titled. **Sign your plot and Matlab code to indicate authorship.**