BOUMAN
NOT TO BE HANDED IN

EE301 Homework #12

Problem 1 *DFT*

For each of the following signals, calculate the DFT X_k for $0 \leq k < N$.

(a) $x[n] = \delta[n]$ for $0 \leq n < N$.

(b) $x[n] = \delta[n-k]$ for $0 \leq n, k < N$.

(c) $x[n] = e^{j\frac{2\pi nm}{N}}$ for $0 \leq n, m < N$.

(d) $x[n] = \cos\left(j\frac{2\pi nm}{N}\right)$ for $0 \leq n, m < N$.

(e) $x[n] = \sin\left(j\frac{2\pi nm}{N}\right)$ for $0 \leq n, m < N$.

(f) $x[n] = u[n-p] - u[n-q]$ for $0 \leq n < N$ and $0 \leq p < q < N$.

Problem 2 *Parseval's Theorem for the DFT*

(a) Let the functions $\phi_k[n]$ for $0 \leq n < N$ have the property that

$$< \phi_k, \phi_l >= \alpha\delta[k-l]$$

and let

$$x[n] = \sum_{k=0}^{N-1} X_k \phi_k[n] \ .$$

Then prove that

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{\alpha} \sum_{k=0}^{N-1} |X_k|^2$$

(b) Define the functions $\phi_k[n]$, the constant α, and the innerproduct $< \phi_k, \phi_l >$ so that the DFT transform (as defined in lecture) has the same structure as described in part a) above.

(c) Prove Parseval's relation for the DFT.

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2$$

Problem 3 *DTFT*

(a) Compute the DTFT, $X(\omega)$, for the following signals.

(a) $x[n] = u[n+m] - u[n-m-1]$ for $m \geq 0$.

(b) $x[n] = \delta[n-k]$

(c) $x[n] = \cos(\omega_0 n + \phi)$

(d) $x[n] = \sin(\omega_0 n + \phi)$

(e) $x[n] = a^n u[n]$ where $|a| < 1$

(e) $x[n] = a^{|n|}$ where $|a| < 1$

(f) $x[n] = na^n u[n]$ where $|a| < 1$

(g) $x[n] = a^{n-1} u[n-1]$ where $|a| < 1$

(h) $x[n] = e^{(j\omega_o - a)n} u[n]$ where $a > 1$

Problem 4 *Difference Equations*

Consider the discrete time system $y[n] = T[x[n]]$ with input $x[n]$ and output $y[n]$ which obeys the following difference equation

$$y[n] = 2r\cos(\theta)y[n-1] - r^2 y[n-2] + x[n]$$

where $|r| < 1$ and θ are real valued constants.

(a) Prove the system $T[\cdot]$ is linear.

(b) Prove the stem $T[\cdot]$ is time invariant.

(c) Calculate the frequency response $H(\omega)$ of the system.

(d) Calculate the impulse response $h[n]$ of the system.

Problem 5 *Sampling and DTFT's*

Consider the functions

$$y[n] = x(nT)$$

For each example, i) sketch $x(t)$, i) calculate $X(\omega)$ the CTFT of $x(t)$, ii) sketch $|X(\omega)|$, iv) sketch $y[n]$, v) calculate $Y(\omega)$ the DTFT of $y[n]$, vi) sketch $|Y(\omega)|$, vii) indicate if there is aliasing.

(a) $x(t) = (\mathrm{sinc}(t))^2$ and $T = 3/8$.

(b) $x(t) = (\mathrm{sinc}(t))^2$ and $T = 1/2$.

(c) $x(t) = (\mathrm{sinc}(t))^2$ and $T = 5/8$.

Problem 4 *Sampling and Reconstruction*

A signal $x(t)$ is sampled at period T to form $y[n]$.

$$y[n] = x(nT)$$

The signal $y[n]$ is then used as the input to an impulse generator to form $s(t)$.

$$s(t) = \sum_{k=-\infty}^{\infty} y[n]\delta(t - kT)$$

The signal $s(t)$ is then filtered to form the final output $z(t)$ using the filter $H(\omega)$.

(a) Sketch a general function $X(\omega)$ which is bandlimited to $|\omega| < \frac{\pi}{T}$.

(b) Calculate $Y(\omega)$ in terms of $X(\omega)$.

(c) Sketch $Y(\omega)$ for a typical function $X(\omega)$.

(d) Calculate $S(\omega)$ in terms of $X(\omega)$.

(e) Sketch $S(\omega)$.

(f) Calculate $Z(\omega)$ in terms of $X(\omega)$.

(g) Calculate $Z(\omega)$ in terms of $X(\omega)$ assuming that $H(\omega) = T\mathrm{rect}(T\omega/(2\pi))$.

(h) Sketch $Z(\omega)$ assuming that $H(\omega) = T\mathrm{rect}(T\omega/(2\pi))$.