EE301 Homework #10

Problem 1 *Duality property of the CTFT*

Use the duality property to determine the CTFT of the following signals

(a) $x(t) = \frac{1}{5+j2\pi t}$

(b) $x(t) = \frac{t}{(1+t^2)^2}$ (Hint: see question 4.12 in the textbook.)

Problem 2 *Symmetry properties of the CTFT*

For each of the following transforms, determine whether the corresponding time-domain signal is (i) real, purely imaginary, or complex, and (ii) even, odd, or neither even nor odd. Do this without evaluating the inverse CTFT.

(a) $X(\omega) = \sin(2\omega)\cos(3\omega)$

(b) $X(\omega) = \sin(\omega)\, e^{j(2\omega+\pi/2)}$

(c) $X(\omega) = u(\omega) - u(\omega - 4\pi)$

Problem 3

Consider a LTI system with frequency response $H(\omega)$, input $x(t)$, and output $y(t)$.

(a) Derive an expression for

$$\int_{-\infty}^{\infty} h(t)dt$$

in terms of the function $H(\omega)$.

(b) Derive an expression for $h(0)$ in terms of $H(\omega)$.

(c) If the input is $x(t) = a$, then express the output $y(t)$ in terms of a and $H(\omega)$.

(d) If the input is $x(t) = a$, then express the output $y(t)$ in terms of a and $h(t)$.

(e) You are asked to design a LTI system with a DC gain of A. What do you know about the impulse response of the system?

(f) You are asked to design a LTI system with a DC gain of A. What do you know about the frequency response of the system?

Problem 4 *Frequency analysis of linear differential equations*

Consider the system with input $x(t)$ and output $y(t)$ described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 10y(t) = \frac{dx(t)}{dt} - x(t)$$

where the system is assumed to be initially at rest.

(a) Determine the frequency response of the system $H(\omega)$.

(b) Determine the impulse response of the system $h(t)$.

(c) If the input to the system is $x(t) = e^{-t}u(t)$ find the corresponding output.