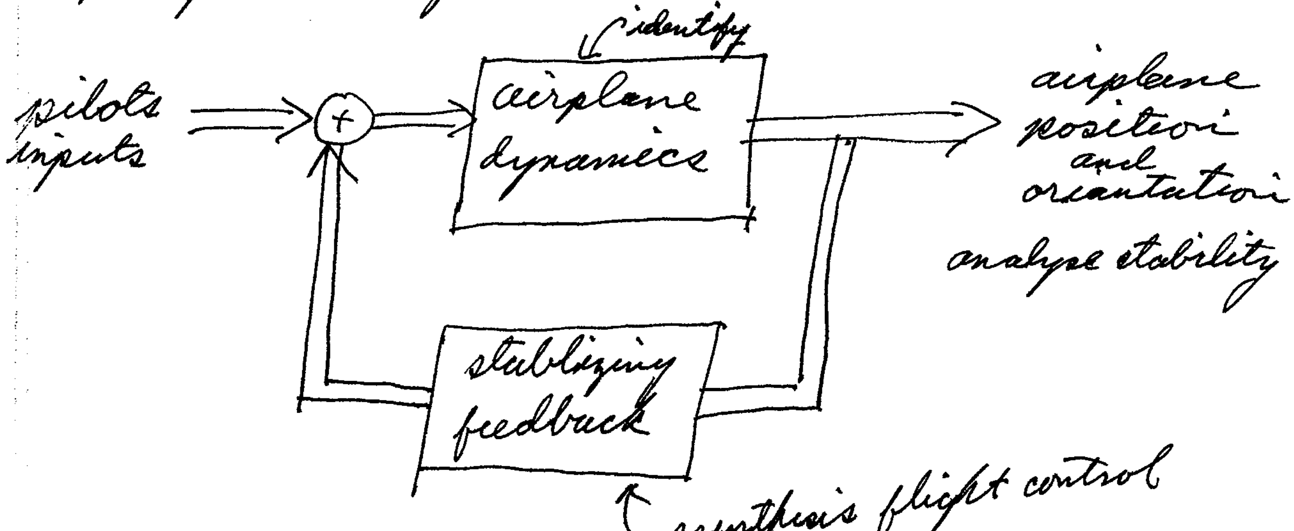
Systems - Relates inputs to outputs

We will mainly deal with the case of one input one output systems.

Principal objectives of systems engineering

1. Identification - Choose a mathematical model which best describes observed inputs and outputs.
2. Analysis - determine the properties of the mathematical model
3. Synthesis or Design - construct a system with a desired behavior

Example: Flight control on the F-16



System Properties

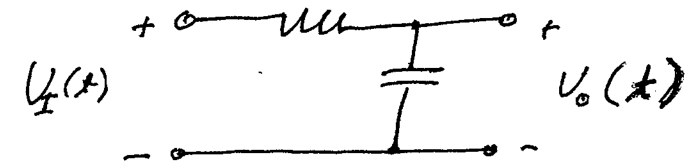
1. continuous/discrete Systems

continuous system - continuoustime input/output

discrete system - discretetime input/output

example:

A. continuous system



$$V_I(t) \longrightarrow \boxed{\begin{matrix}\text{system} \\ S\end{matrix}} \longrightarrow V_0(t)$$

$$V_0(t) = S[V_I(t)]$$

For every input signal $V_I(t)$ there is an output signal $V_o(t)$

B. Discrete system

$$Y_K = Y_{K-1} + X_K$$

$$X_K \longrightarrow \boxed{\begin{matrix}\text{system} \\ S\end{matrix}} \longrightarrow Y_K$$

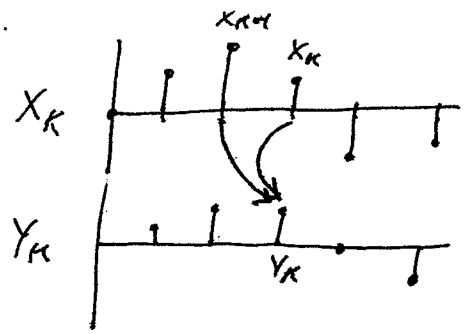
$$Y_K = S[X_K]$$

2. Causal / noncausal Systems

causal - only uses the past and present inputs

noncausal - ~~may~~ also uses future ~~data~~ inputs

example: $Y_K = X_K + X_{K-1}$ causal



$Y_K = X_{K+1}$ noncausal

3. Memory and Memoryless Systems

Memoryless - only used the present input

System with Memory - used past and/or future input

example:

$$Y(t) = (x(t))^2 \quad \text{memoryless}$$

$$Y(t) = \int_{t-1}^{t} x(\tau)\, d\tau \quad \text{memory}$$

4. Linearity (very important)

A linear system must have two properties

Homogeneity – forall α

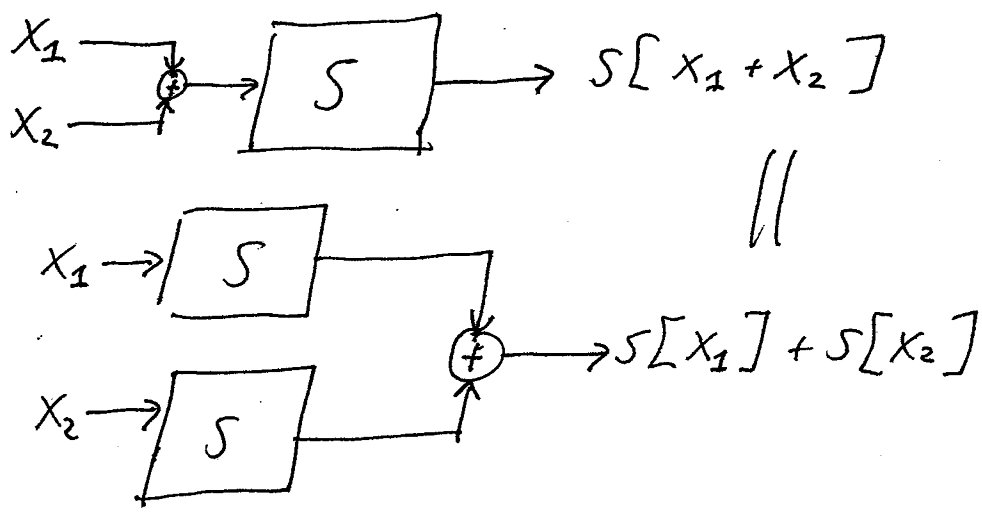
$$S[\alpha x(t)] = \alpha S[x(t)]$$

$$\text{or } S[\alpha x_n] = \alpha S[x_n]$$

Double input $\Rightarrow$ Double output

Superposition –

$$S[x_1(t) + x_2(t)] = S[x_1(t)] + S[x_2(t)]$$



Definition: A system S is linear if for any two input signals $x_1(t)$ $x_2(t)$ and any two constants ~~[illegible]~~ α, β

$$S[\alpha x_1(t) + \beta x_2(t)] = \alpha S[x_1(t)] + \beta S[x_2(t)]$$

Key idea - we can break up complex signals into simple component signals and then add the individual solutions

example: $y(t) = \int_0^t x(\tau)\,d\tau = S[x(t)]$

$$y_1(t) = S[x_1(t)] = \int_0^t x_1(\tau)\,d\tau$$
$$y_2(t) = S[x_2(t)] = \int_0^t x_2(\tau)\,d\tau$$

$$S[\alpha x_1(t) + \beta x_2(t)] = \int_0^t [\alpha x_1(\tau) + \beta x_2(\tau)]\,d\tau$$
$$= \alpha \int_0^t x_1(\tau)\,d\tau + \beta \int_0^t x_2(\tau)\,d\tau$$
$$= \alpha y_1(t) + \beta y_2(t)$$
$$= \alpha S[x_1(t)] + \beta S[x_2(t)]$$

<u>Linear</u>

example: $y(t) = x(t) + c$

$$S[\alpha x_1(t) + \beta x_2(t)] = \alpha x_1(t) + \beta x_2(t) + c$$

$$\overset{?}{=} \alpha S[x_1(t)] + \beta S[x_2(t)] = \alpha x_1(t) + c + \beta x_2(t) + c$$

equality $\Leftrightarrow$ $c = 0$ $\boxed{\text{nonlinear for } c \neq 0}$

If S is linear:

$$S\left[\sum_{i=1}^{N} \alpha_i X_i(t)\right] = S\left[\sum_{i=1}^{N-1} \alpha_i X_i(t) + \alpha_N X_N(t)\right]$$

$$= S\left[\sum_{i=1}^{N-1} \alpha_i X_i(t)\right] + \alpha_N S\left[X_N(t)\right]$$

By mathematical induction

$$\boxed{S\left[\sum_{i=1}^{N} \alpha_i X_i(t)\right] = \sum_{i=1}^{N} \alpha_i S\left[X_i(t)\right]}$$

5. Time-Invariant and Time-Varying Systems

A continuous time system is time-invariant if for all constants T

$$\text{if} \quad y(t) = S[X(t)]$$
$$\text{then} \quad y(t-T) = S[X(t-T)]$$

(or equivalently $S[X(t)] = S[X(t-T)]$)

A discrete time system is time-invariant if for all intergers K

$$\text{if} \quad y(n) = S[X(n)]$$
$$\text{then} \quad y(n-K) = S[X(n-K)]$$

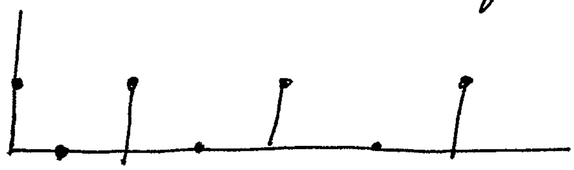
(equivalently $S[X(n)] = S[X(n-K)]$)

In words — delaying the input simply delays the output.

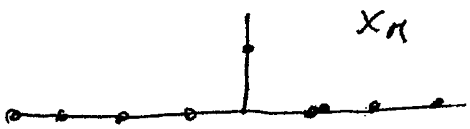
example: $y_K = \alpha_K x_K$

$$\alpha_K = \begin{cases} 1 & \text{if } K \text{ is even} \\ 0 & \text{if } K \text{ is odd} \end{cases}$$

α_K



$$x_K = \begin{cases} 1 & \text{if } K=0 \\ 0 & \text{otherwise} \end{cases}$$



$$y_K = \alpha_K x_K = x_K \neq 0$$

$$0 \neq y_{K-1} \overset{?}{=} \alpha_K x_{K-1} = 0$$

$$\boxed{\text{time varying}}$$

6. Stable and unstable systems

- bounded input / bounded output (BIBO) stability.

Definitions: A system is BIBO stable if for all input signals $x(t)$ such that $|x(t)|$ is bounded the $y(t) = T[x(t)]$ is bounded.

examples:

1) $$y(t) = e^{at} x(t)$$

$\Rightarrow$ not stable

2) $$y(t) = \exp\{x(t)\}$$

$\Rightarrow$ stable

Example: Median filter

1) describe
2) check for properties of
   a) Continuous/discrete time
   b) causal/noncausal
   c) Memory/memoryless
   d) Stable/Unstable
   e) Invertible/Noninvertible
   f) Time invariant/time varying
   g) Linear
      i) homogeneous
      ii) superposition