Some useful formulae

Some useful formulae for linear systems

If $h(t,\sigma)$ is the impulse response of a linear system, then for any input $x(t);\ -\infty < t < \infty$

$$y(t) = \int_{-\infty}^{\infty} h(t,\sigma)x(\sigma)d\sigma$$

If the system is causal then $h(t,\sigma) = h(t,\sigma)U(t-\sigma)$.

If the system is time-invariant then $h(t,\sigma) = h(t-\sigma)$.

The I/O relationship for linear, time-invariant and causal systems subject to causal inputs $x(t)$ with $x(t) = 0,\ t < 0$ is given by:

$$y(t) = \int_0^t h(t-\sigma)x(\sigma)d\sigma = \int_0^t h(\sigma)x(t-\sigma)d\sigma; \qquad t \geq 0$$

Leibniz's Rule for Differentiation of an Integral

Assume that $f(t,\tau)$ is differentiable w.r.t the first variable and $a(t)$ and $b(t)$ are differentiable functions. Then:

$$\frac{d}{dt}\left(\int_{a(t)}^{b(t)} f(t,\tau)d\tau\right) = f(t,b(t))\frac{db(t)}{dt} - f(t,a(t))\frac{da(t)}{dt} + \int_{a(t)}^{b(t)} \frac{\partial f(t,\tau)}{\partial t}d\tau$$

Some useful Laplace transforms

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Function	Laplace Transform
$\delta(t)$	1
U(t)	$\frac{1}{s}$
$e^{\pm at}U(t)$	$\frac{1}{s \mp a}$
$\sin(at)U(t)$	$\frac{a}{a^2+s^2}$
$\cos(at)U(t)$	$\frac{s}{s^2+a^2}$
$\frac{df(t)}{dt}$	$sF(s) - f(0+)$
$\int_0^t f(u)du$	$\frac{F(s)}{s}$
$\int_0^t f(t-u)g(u)du$	$F(s)G(s)$
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$