1 Signal Properties

Below are some formal signal properties that we will use. Continuous-time (CT) signals are functions from the reals, $\Re$, which take on real values; and discrete-time (DT) signals are functions from the integers $\mathbb{Z}$, which take on real values.

Definition *A continuous time signal is* **bounded** *if there exists an M such that for all $t \in \Re$*

$$|x(t)| \leq M \ .$$

Definition *A discrete time signal is* **bounded** *if there exists an M such that for all $n \in \mathbb{Z}$*

$$|x[n]| \leq M \ .$$

Definition *A signal is* **unbounded** *if it is not bounded.*

Definition *A CT signal is* **absolutely integrable** *if*

$$\int_{-\infty}^{\infty} |x(t)| < \infty$$

Definition *A DT signal is* **absolutely summable** *if*

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

Definition *A CT or DT signal is said to be* **stable** *if it is absolutely integrable or summable.*

Definition *A CT signal is* **causal** *if* for all $t < 0$, $x(t) = 0$.

Definition *A DT signal is* **causal** *if* for all $n < 0$, $x[n] = 0$.

Definition *A CT signal is* **right-sided** *if* there exists a T such that for all $t < T$, $x(t) = 0$.

Definition *A DT signal is* **right-sided** *if* there exists a T such that for all $n < T$, $x[n] = 0$.

Definition *A CT signal is* **left-sided** *if* there exists a T such that for all $t > T$, $x(t) = 0$.

Definition *A DT signal is* **left-sided** *if* there exists a T such that for all $n > T$, $x[n] = 0$.

Definition *A CT or DT signal is said to be* **two-sided** *if it is not left-sided and it is not right-sided.*

2 System Properties

In general, a system is an operator that takes a function as its input and produces a function as its output. A discrete-time (DT) system has inputs and outputs that are discrete-time functions, and a continous-time (CT) system has inputs and outputs that are continous-time functions.

So for example, let $S[\cdot]$ denote a DT system that has DT input $x[n]$ and discrete time output $y[n]$. Then we denote this input/output relationship by

$$y[n] = S\left[x[n]\right]$$

where we emphasize that $S[\cdot]$ operates on the entire input signal $x[n]$ to produce the output signal $y[n]$.

For a CT system we use the notation

$$y(t) = S\left[x(t)\right]$$

where $x[n]$ is the CT input and $y[n]$ is the CT output.

Below are definitions for the basic properties of DT and CT systems.

2.1 Memoryless Systems

Definition *A continuous time system $S[\cdot]$ is* **memoryless** *if for all t there exists a function $f_t(\cdot)$ with scalar input and output such that*

$$y(t) = f_t(x(t))$$

where $x(t)$ and $y(t)$ are the systems input and output related by

$$y(t) = S\left[x(t)\right] \ .$$

Definition *A discrete time system $S[\cdot]$ is* **memoryless** *if for all n there exists a function $f_n(\cdot)$ with scalar input and output such that*

$$y[n] = f_n(x[n])$$

where $x[n]$ and $y[n]$ are the systems input and output related by

$$y[n] = S\left[x[n]\right] \ .$$

Definition *A system is* **has memory** *if it is not memoryless.*

2.2 Causal System

Definition *A continuous time system $S[\cdot]$ is* **causal** *if for all T and for all functions $x_1(t)$ and $x_2(t)$ such that for all $t \leq T$, $x_1(t) = x_2(t)$ then*

$$y_1(T) = y_2(T)$$

where $x_1(t)$, $x_2(t)$ and $y_1(t)$, $y_2(t)$ are the system inputs and outputs related by

$$\begin{aligned} y_1(t) &= S\left[x_1(t)\right] \\ y_2(t) &= S\left[x_2(t)\right] \ . \end{aligned}$$

Definition *A discrete time system $S[\cdot]$ is* **causal** *if for all K and for all functions $x_1[n]$ and $x_2[n]$ such that for all $n \leq K$, $x_1[n] = x_2[n]$ then*

$$y_1[K] = y_2[K]$$

where $x_1[n]$, $x_2[n]$ and $y_2[n]$, $y_2[n]$ are the system inputs and outputs related by

$$\begin{aligned} y_1[n] &= S\left[x_1[n]\right] \\ y_2[n] &= S\left[x_2[n]\right] \ . \end{aligned}$$

Definition *A system is* **noncausal** *if it is not causal.*

2.3 Linearity

Definition *A continuous time system $S[\cdot]$ is* **linear** *if for all α and for all β and for all functions $x_1(t)$ and $x_2(t)$ the following is true*

$$S\left[\alpha x_1(t) + \beta x_2(t)\right] = \alpha S\left[x_1(t)\right] + \beta S\left[x_2(t)\right] \ .$$

Definition *A discrete time system* $S[\cdot]$ *is* **linear** *if*
for all α *and for all* β *and for all functions* $x_1[n]$ *and* $x_2[n]$ *the following is true*

$$S\left[\alpha x_1[n] + \beta x_2[n]\right] = \alpha S\left[x_1[n]\right] + \beta S\left[x_2[n]\right] \ .$$

Definition *A system is* **nonlinear** *if it is not linear.*

2.4 Time Invariance

Definition *A continuous time system* $S[\cdot]$ *is* **time invariant** *if*
for all d *and for all functions* $x(t)$ *and for* $y(t) = S\left[x(t)\right]$ *then it is always the case that*

$$y(t-d) = S\left[x(t-d)\right] \ .$$

Definition *A discrete time system* $S[\cdot]$ *is* **time invariant** *if*
for all integers K *and for all functions* $x[n]$ *and for* $y[n] = S\left[x[n]\right]$ *then it is always the case that*

$$y[n-K] = S\left[x[t-K]\right] \ .$$

Definition *A system is* **time varying** *if it is not time invariant.*

2.5 Stability

Definition *A continuous time system* $S\left[\cdot\right]$ *is* **bouned-input-bounded-output (BIBO) stable** *if*
for all bounded input functions $x(t)$ *the output function* $y(t) = S\left[x(t)\right]$ *is also bounded.*

Definition *A discrete time system* $S\left[\cdot\right]$ *is* **bouned-input-bounded-output (BIBO) stable** *if*
for all bounded input functions $x[n]$ *the output function* $y[n] = S\left[x[n]\right]$ *is also bounded.*

Definition *A system is* **BIBO unstable** *if it is not BIBO stable.*