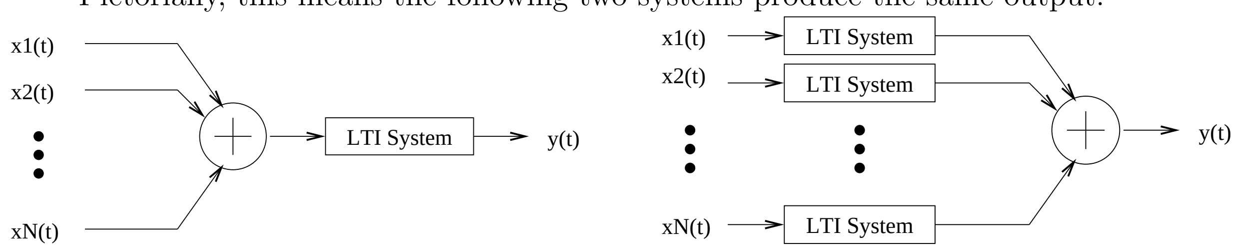
Dr. Bouman's Easy 5-Step Program to Success in EE301

During the semester, we will learn 5 essential concepts in EE301. These concept are very powerful because they allow all linear time invariant (LTI) systems to be easily analyzed. These concepts are at the core of most Electrical Engineering problems, and are found in many other disciplines.

1. **All LTI systems obey superposition:** Superposition is part of the definition of a linear system, so all LTI systems have this property. It means that the response of an LTI system to a sum of signals is equal to the sum of the responses of the system to each individual signal.

   Pictorially, this means the following two systems produce the same output.



2. **The output of an LTI system is given by the convolution of the input with the system's impulse response:**
   Continuous-time case:
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$
   Discrete-time case:
$$y_n = h_n * x_n = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$$

3. **A sine wave input to an LTI system produces a sine wave output, but with a different amplitude and phase:**
   For a CT system with complex input, this means:
$$Ae^{j(\omega t+\phi)} = \mathcal{S}\left[e^{j\omega t}\right]$$
   For a CT system with real input, this means:
$$A\sin(\omega t+\phi) = \mathcal{S}\left[\sin(\omega t)\right]$$
   For a DT system with complex input, this means:
$$Ae^{j(\omega n+\phi)} = \mathcal{S}\left[e^{j\omega n}\right]$$
   For a DT system with real input, this means:
$$A\sin(\omega n+\phi) = \mathcal{S}\left[\sin(\omega n)\right]$$
   In each case, A and ϕ are real valued constants that may change for each frequency ω.

4. **Any signal can be represented as a sum of sine waves:**
This is the essence of the Fourier transform. For continuous-time, nonperiodic signals:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

For discrete-time, nonperiodic signals:

$$x_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

For each frequency ω, $X(\omega)$ is the complex number that controls the amplitude and phase of the output sine wave.

5. **Any LTI system can be analyzed by separating the input into sine waves, finding the response of the system to each sine wave, and adding the individual reponses together:**
This is called frequency domain analysis of a system. This concept results from putting together concepts 1, 3, and 4 above. Mathematically, if the input to a system is $x(t)$ and the output is $y(t)$, then we know that by concept 4 above, we can represent both the input and output by sums of sine waves.

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \\ y(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega \end{aligned}$$

However, that by concepts 1 and 3 above, we can represent the output at each input frequency by gain and phase shift. I

$$\begin{aligned} Y(\omega) &= A(\omega)e^{j\phi(\omega)}X(\omega) \\ &= H(\omega)X(\omega) \end{aligned}$$

where $A(\omega)$ is the amplitude as a function of frequency, $\phi(\omega)$ is the phase as a function of frequency, and $H(\omega) = A(\omega)e^{j\phi(\omega)}$ is the complex number known as the **transfer function** of the system.