1a.

P	Q	$P \Rightarrow Q$	$\neg P \vee Q$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

1b.

P	Q	$\neg(P \Rightarrow Q)$	$\neg Q \,\&\, P$
1	1	0	0
1	0	1	1
0	1	0	0
0	0	0	0

1c.

P	Q	$\neg(P \vee Q)$	$\neg P \,\&\, \neg Q$
1	1	0	0
1	0	0	0
0	1	0	0
0	0	1	1

1d.

P	Q	$\neg(P \,\&\, Q)$	$\neg P \vee \neg Q$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	1	1

2a. $\forall x \in A,\ Px$

i). $\neg(\forall x \in A,\ Px) = \exists x \in A$ s.t. $\neg Px$

ii). $A = \{\text{Balls}\}$

Px = x is blue.

iii) All balls are blue.

b. $\forall x \in A,\ Px \Rightarrow Qx$

i). $\neg(\forall x \in A,\ Px \Rightarrow Qx) = \exists x \in A$ s.t. $\neg Q \,\&\, P$

ii) $A = \{\text{Balls}\}$

Px = x is a basketball

Qx = bounces

iii) All balls that are basketballs bounce.

c. $\exists x \in A$ s.t. $(\neg Px \vee Qx)$

i). $\neg(\exists x \in A$ s.t. $(\neg Px \vee Qx)) = \forall x \in A,\ P \,\&\, \neg Q$

ii) $A = \{\text{toys}\}$

Px = x is a ball

Qx = x is bouncy.

iii) There exists a toy that is not a ball or is bouncy.

2d. $\forall x \in A\ \exists y \in B$ s.t. P_{xy}.

i). $\exists x \in A$ s.t. $\forall y \in B$, $\neg P_{xy}$.

ii) A = { jobs }

B = { people }

P_{xy} = Person y likes jobs x.

iii) For all jobs, there is a person who will like that job.

---

e. $\exists x \in A$ s.t. $\forall y \in B$, P_{xy}.

i) $\forall x \in A$, $\exists y \in B$ s.t. $\neg P_{xy}$.

ii) A: { Students at Purdue }

B: { different types of sports }

P_{xy} : Student x likes sport y.

iii) There is a student at Purdue who likes all types of sports.

---

3) "A signal $x(t)$ is bounded if there exists an $M>0$ such that

$|x(t)| < M$ for all t."

a: $\exists M>0$ st. $\forall t \in \mathbb{R}$ $|x(t)| < M \Leftrightarrow$ bounded

b: $\forall M>0$ $\exists t \in \mathbb{R}$ s.t. $|x(t)| \geq M \Leftrightarrow$ unbounded

c: Definition of unbounded: "For all values of $M>0$, we can find a value of $t \in \mathbb{R}$ such that $|x(t)| > M$"

d: Let $M=2$

For all $t \in \mathbb{R}$, $|\sin(t)| \leq 1 < 2 = M$

Thus $|x(t)| < M$ & $x(t)$ is bounded

e: For any value of $M>0$, let $t = 2M$

Then $|x(t)| = |t| = 2M \geq M$

thus $|x(t)| \geq M$ & is unbounded

4a.

i). $S[x_1(t)] = u(t)\, x_1(t)$

$S[x_2(t)] = u(t)\, x_2(t)$.

$x_3(t) \triangleq = a x_1(t) + b x_2(t)$

$$
\begin{aligned}
S[x_3(t)] &= u(t)\, x_3(t) \\
&= u(t)\big(a x_1(t) + b x_2(t)\big) \\
&= a\, u(t)\, x_1(t) + b\, u(t)\, x_2(t)
\end{aligned}
$$

which is equal to.

$a S[x_1(t)] + b S[x_2(t)] = a\, u(t)\, x_1(t) + b\, u(t)\, x_2(t)$.

Since $S[a x_1(t) + b x_2(t)] = a S[x_1(t)] + b S[x_2(t)]$

the system S is linear.

ii. let $y(t) = S[x(t)]$

$y(t) = u(t)\, x(t)$

$y(t-T) = u(t-T)\, x(t-T)$.

compare to.

$S[x(t-T)] = u(t)\, x(t-T)$.

since $y(t-T) \neq S[x(t-T)]$

the system S is time-varying.

Counter example.

$x(t) = u(t)$, $T = -1$.

$S[x(t)] = y(t) = u(t)$.

$S[x(t+1)] = u(t)\, x(t+1) = u(t)$.

$y(t+1) = u(t+1)$

$S[x(t+1)] \neq y(t+1)$.

iii). The system depends only on present time. It's causal.

iv). The system is memoryless because it depends only on inputs at the present time.

b. $y(t) = x(\sin(t))$.

i).

$S[x_1(t)] = x_1(\sin(t))$

$S[x_2(t)] = x_2(\sin(t))$.

$x_3(t) \triangleq a x_1(t) + b x_2(t)$

$$\begin{aligned} S[x_3(t)] &= x_3(\sin(t)) \\ &= a\, x_1(\sin(t)) + b\, x_2(\sin(t)). \end{aligned}$$

compare to.

$a S[x_1(t)] + b S[x_2(t)] = a\, x_1(\sin(t)) + b\, x_2(\sin(t))$.

Since $S[a x_1(t) + b x_2(t)] = a S[x_1(t)] + b S[x_2(t)]$

the system S is linear.

ii). Let $y(t) = S[x(t)]$

$y(t) = x(\sin(t))$

$y(t-T) = x(\sin(t-T))$

$S[x(t-T)] = x(\sin(t)-T)$

Since $S[x(t-T)] \neq y(t-T)$,

the system S is time-varying.

A counter example :

$x(t) = \sin^{-1}(t)$, $T=1$.

$y(t) = s[x(t)] = t$

$s[x(t-1)] = \sin^{-1}(\sin(t)-1)$

$s[x(t-1)] \neq y(t-1)$

Is there any time $t < \sin(t)$? If yes, S is non-causal.

iii) Since $-1 \leq \sin(t) \leq 1$, so for any $t<-1$, $\sin(t) > t$.

Therefore, the system is non causal.

iv) Since $\sin(t)$ is not alway equal to t,

S has memory.

(C) $y(t) = \sin(x(t))$.

i). $S[x_1(t)] = \sin(x_1(t))$

$S[x_2(t)] = \sin(x_2(t))$

$x_3(t) \triangleq a x_1(t) + b x_2(t)$

$s[x_3(t)] = \sin(x_3(t))$

$= \sin(a x_1(t) + b x_2(t))$

compare to

$a S[x_1(t)] + b s[x_2(t)] = a \sin(x_1(t)) + b \sin(\ _2(t))$

since $s[a x_1(t) + b x_2(t)] \neq a s[x_1(t)] + b s[x_2(t)]$

the system S is not linear.

counter example.

$x_1(t) = \sin^{-1}(t)$, $x_2(t) = 0$.

$s[x_1(t)] = t$.

$x_3(t) = a x_1(t)$.

$s[x_3(t)] = \sin(a \sin^{-1}(t))$

but $a s[x_1(t)] = at$ ∴ S is not linear ii Let $y(t) = S[x(t)]$

$y(t) = \sin(x(t))$.

$y(t-T) = \sin(x(t-T))$

$S[x(t-T)] = \sin(x(t-T))$.

since $y(t-T) = S[x(t-T)]$ the system is T.I.

iii. The system is causal because it depends only on present input.

iv. The system is memoryless because it depends only on presnt input.

d) $y(t) = \frac{dx(t)}{dt}$

i). $S[x_1(t)] = \frac{dx_1(t)}{dt}$

$S[x_2(t)] = \frac{dx_2(t)}{dt}$

$x_3(t) = ax_1(t) + bx_2(t)$.

$S[x_3(t)] = \frac{d}{dt}(ax_1(t) + bx_2(t))$

$= a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt}$

compare to

$aS[x_1(t)] + bS[x_2(t)] = a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt}$.

Since $S[ax_1(t) + bx_2(t)] = aS[x_1(t)] + bS[x_2(t)]$

the system S is linear.

ii). Let $y(t) = S[x(t)]$

$$y(t) = \frac{dx(t)}{dt}.$$

$$y(t-T) = \frac{dx(t-T)}{dt}.$$

$$S[x(t-T)] = \frac{dx(t-T)}{dt}$$

Since $y(t-T) = S[x(t-T)]$
the system S is TI.

iii) The system is causal because it does not depend on future inputs.

iv). The system is memoryless because it depends only on input at the present time.

e) $y(t) = x(2t) - x(t-1)$.

i) $S[x_1(t)] = x_1(2t) - x_1(t-1)$
$S[x_2(t)] = x_2(2t) - x_2(t-1)$
$x_3 \triangleq a x_1(t) + b x_2(t)$
$S[x_3(t)] = a x_1(2t) + b x_2(2t) - a x_1(t-1) - b x_2(t-1)$

Compare to.

$$a S[x_1(t)] + b S[x_2(t)] = a\left(x_1(2t) - x_1(t-1)\right) + b\left(x_2(2t) - x_2(t-1)\right).$$

$$= a x_1(2t) + b x_2(t) - a x_1(t-1) - b x_2(t-1).$$

Since $a S[x_1(t)] + b S[x_2(t)] = S[a x_1(t) + b x_2(t)]$
the system S is linear

4.e ii) Let $y(t) = S[x(t)] = x(2t) - x(t-1)$.

compare shifted output

$y(t-T) = x(2(t-T)) - x(t-T-1)$

to system response to shifted input.

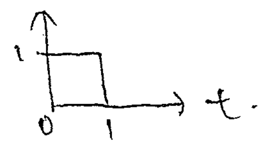
$S[x(t-T)] = x(2t-T) - x(t-T-1)$.

since $y(t-T) \neq S[x(t-T)]$

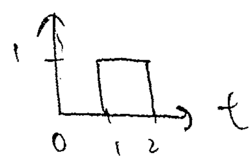
the system is time-varying.

Counter example.

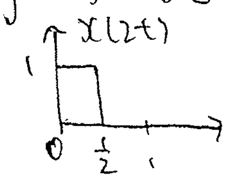
$x(t)$



shift input (by 1) $x(t-1)$.



$y(t) = S[x(t)] = x(2t) - x(t-1)$.



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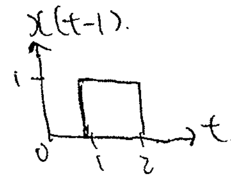
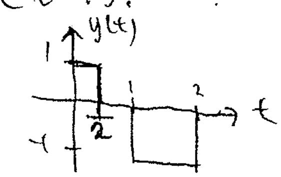
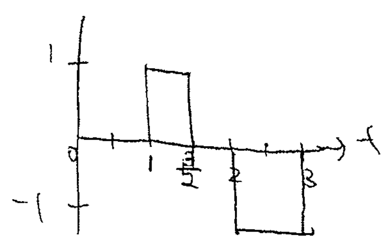


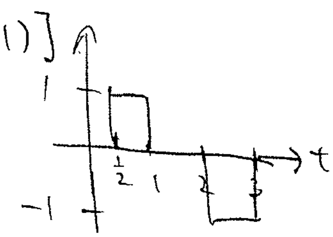
$=$



shifted output. $y(t-1) =$



but $S[x(t-1)]$



$\neq y(t-T)$

iii). if $t > 0$, $2t > t$

So S depends on future inputs. S is non-causal.

iv. S has memory.

4 (f) $y(t) = x(0)$.

(i). $S[x_1(t)] = x_1(0)$

$S[x_2(t)] = x_2(0)$.

$x_3(t) \triangleq a x_1(t) + b x_2(t)$.

$S[x_3(t)] = a x_1(0) + b x_2(0)$.

Compare to

$a S[x_1(t)] + b S[x_2(t)] = a x_1(0) + b x_2(0)$

Since $S[a x_1(t) + b x_2(t)] = a S[x_1(t)] + b S[x_2(t)]$

the system S is linear.

ii. let $y(t) = S[x(t)]$

$y(t) = x(0)$.

$y(t-T) = x(0)$.

$S[x(t-T)] = x(-T)$.

Since $y(t-T) \neq S[x(t-T)]$,

the system is time-varying.

counter example.

$x(t) = u(t-1)$ and $T = -2$.

$y(t) = S[x(t)] = x(0) = 0$

$S[x(t+2)] = S[u(t+1)] = 1$

$\underbrace{y(t+2)}_{=0} \neq S[x(t+2)]$

iii. The system is non-causal because the output depends on future input for all $t < 0$.

iv. The system has memory.

4.9. $y(t) = \int_0^t x(\tau)\,d\tau$.

i). $S[x_1(t)] = \int_0^t x_1(\tau)\,d\tau$

$S[x_2(t)] = \int_0^t x_2(\tau)\,d\tau$.

$x_3(t) \triangleq a_1 x_1(t) + b\, x_2(t)$

$S[x_3(t)] = a\int_0^t x_1(\tau)\,d\tau + b\int_0^t x_2(\tau)\,d\tau$.

compare to

$a\,S[x_1(t)] + b\,S[x_2(t)] = a\int_0^t x_1(\tau)\,d\tau + b\int_0^t x_2(\tau)\,d\tau$.

since they are equal.

the system is linear.

ii). let $y(t) = S[x(t)]$

$y(t) = \int_0^t x(\tau)\,d\tau$

$y(t-T) = \int_0^{t-T} x(\tau)\,d\tau$.

$S[x(t-T)] = \int_0^t x(\tau-T)\,d\tau$.

let $u = \tau - T$, $du = d\tau$

$S[x(t-T)] = \int_{-T}^{t-T} x(u)\,du$.

$\neq y(t-T)$.

The system is time varying.

Counter example.

$x(t) = u(t)$, $T = -2$.

$x(t+2) = u(t+2)$.

$y(t) = \int_0^t d\tau = t$.

$S[x(t+2)] = \int_0^t u(t+2)\,d\tau = t$.

$y(t+2) = t+2 \neq S[x(t+2)]$.

4g iii) the system is causal since it depends only on current and past inputs.

iv). the system has memory.

5 a). $y_n = x_n + 1$.

i). $S[x_{1n}] = x_{1n} + 1$

$S[x_{2n}] = x_{2n} + 1$.

$x_{3n} \triangleq a\, x_{1n} + b\, x_{2n}$.

$S[x_{3n}] = (a\, x_{1n} + b\, x_{2n}) + 1$

compare to.

$a\, S[x_{1n}] + b\, S[x_{2n}] = x_{1n} + x_{2n} + 2$.

$\neq S[x_{3n}]$

the system is non-linear.

Counter examples.

$x_{1n} = u_n$,

$x_{2n} = u_n$.

$S[x_{1n}] = u_n + 1$

$S[x_{2n}] = u_n + 1$.

$x_{3n} = x_{1n} + x_{2n} = 2u_n$.

$S[x_{3n}] = 2u_n + 1$.

but $S[x_{1n}] + S[x_{2n}] = 2u_n + 2$.

$\neq S[x_{3n}]$

b). $y_n = x(2n)$

i). $S[x_{1n}] = x_1(2n)$

$S[x_{2n}] = x_2(2n)$.

$x_{3n} \triangleq a\, x_{1n} + b\, x_{2n}$.

b(i) cont. $S[x_{3n}] = a\,x_1(2n) + b\,x_2(2n)$.

$a\,S[x_{1n}] + b\,S[x_{2n}] = a\,x_1(2n) + b\,x_2(2n)$.

since $S[x_{3n}] = a\,S[x_{1n}] + b.\,S[x_{2n}]$

the system is linear.

(ii) let $y_n = S[x_n]$

$y_n = x(2n)$.

$y_{n-N} = x_{2(n-N)}$

$S[x_{n-N}] = x_{2n-N} \neq y_{n-N}$.

the system is time-varying.

conter example.

$x_n = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd.} \end{cases}$ $\quad x_{n-1} = \begin{cases} 1, & n \text{ odd} \\ 0, & n \text{ even.} \end{cases}$

$y_n = 1$

but $S[x_{n-1}] = 0. \neq (y_{n-1} = 1)$.

$$= a \begin{cases} x_1[\frac{n}{2}] & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases} + b \begin{cases} x_2[\frac{n}{2}] & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases}$$ (13)

Compare to

$$a S[x_{1n}] + b S[x_{2n}] = a \begin{cases} x_1[\frac{n}{2}] & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases} + b \begin{cases} x_2[\frac{n}{2}] & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$$

Since $S[x_{3n}] = a S[x_{1n}] + b S[x_{2n}]$
the system is linear.

(ii). let $y_n = S[x_n]$.

$$y_n = \begin{cases} x[\frac{n}{2}] & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$$

$$y_{n-N} = \begin{cases} x[\frac{n-N}{2}] & , n-N \text{ even} \\ 0 & , n-N \text{ odd.} \end{cases}$$

$$S[x_{n-N}] = \begin{cases} x[\frac{n}{2}-N] & , n \text{ even.} \\ 0 & , n \text{ odd.} \end{cases}$$

since $y_{n-N} \neq S[x_{n-N}]$,
the system S is time-varying.

counter example.

$x_1[n] = n$

$x_2[n] = x_1[n-1] = n-1$, $N=1$

$$y_1[n] = \begin{cases} \frac{n}{2} & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases} \qquad y_2[n] = \begin{cases} \frac{n}{2}-1 & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases}$$

$$y_1[n-1] = \begin{cases} \frac{n-1}{2} & , n-1 \text{ even} \\ 0 & , n-1 \text{ odd.} \end{cases} \neq y_2[n].$$

(14)

5 d. $y[n] = \begin{cases} x[n], & x[n] < 4 \\ 4, & \text{otherwise} \end{cases}$

i). $S[x_1[n]] = \begin{cases} x_1[n], & x_1[n] < 4 \\ 4, & \text{otherwise} \end{cases}$

$S[x_2[n]] = \begin{cases} x_2[n], & x_2[n] < 4 \\ 4, & \text{otherwise} \end{cases}$

$x_3[n] \triangleq a x_1[n] + b x_2[n]$.

$S[x_3[n]] = \begin{cases} a x_1[n] + b x_2[n], & (a x_1[n] + b x_2[n]) < 4 \\ 4, & \text{otherwise} \end{cases}$

$\neq a S[x_1[n]] + b S[x_2[n]]$

Since $S[a x_1[n] + b x_2[n]] \neq a S[x_1[n]] + b S[x_2[n]]$

the system is non-linear

example.

$x_1[n] = u[n]$

$x_2[n] = 5u[n]$.

$y_1[n] = S[x_1[n]] = u[n]$.

$y_2[n] = S[x_2[n]] = 4u[n]$.

$S[5x_1[n]] \neq 5 S[x_1[n]]$

ii). let $y[n] = S[x[n]]$.

$y[n] = \begin{cases} x[n], & x[n] < 4 \\ 4, & \text{otherwise} \end{cases}$

$y[n-N] = \begin{cases} x[n-N], & x[n-N] < 4 \\ 4, & \text{otherwise} \end{cases}$

$S[x[n-N]] = \begin{cases} x[n-N], & x[n-N] < 4 \\ 4, & \text{otherwise} \end{cases}$

Since $y[n-N] = S[x[n-N]]$

the system is TI