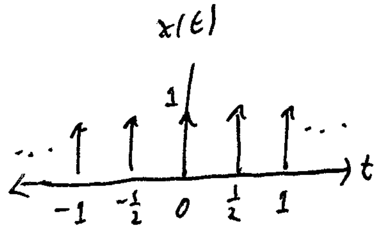
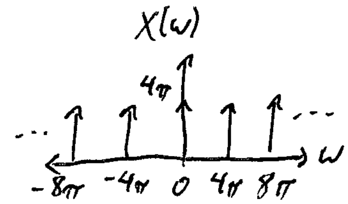
ECE 301 Hw 10 Solutions

1) Note: $\sum_{k=-\infty}^{\infty} \delta(t-kT) \Longleftrightarrow \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$

a: $x(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{1}{2}k\right)$ $\quad T = \frac{1}{2}$

$X(\omega) = 4\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 4\pi k)$



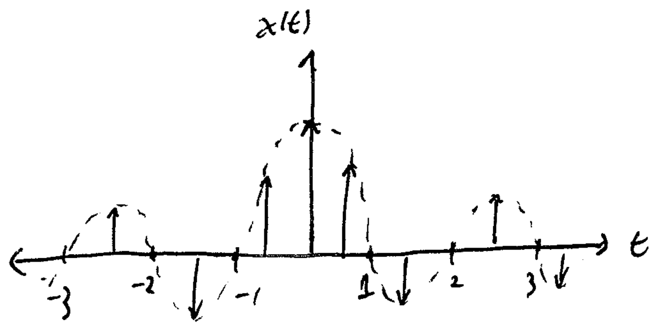


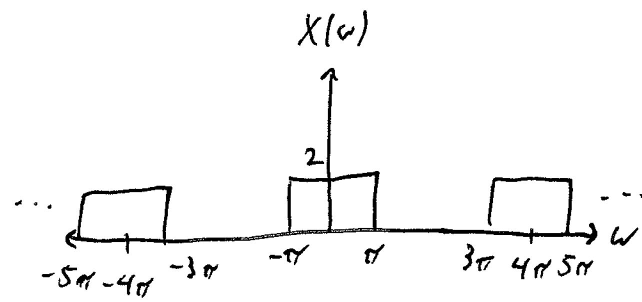
b: $x(t) = \text{sinc}(t) \sum_k \delta\left(t - \frac{1}{2}k\right)$ $\quad$ Note: $\text{sinc}(t) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$

$X(\omega) = \frac{1}{2\pi} \mathcal{F}\{\text{sinc}(t)\} * \mathcal{F}\left\{\sum_k \delta\left(t - \frac{1}{2}k\right)\right\}$

$= \frac{1}{2\pi} \text{rect}\left(\frac{\omega}{2\pi}\right) * 4\pi \sum_k \delta(\omega - 4\pi k)$

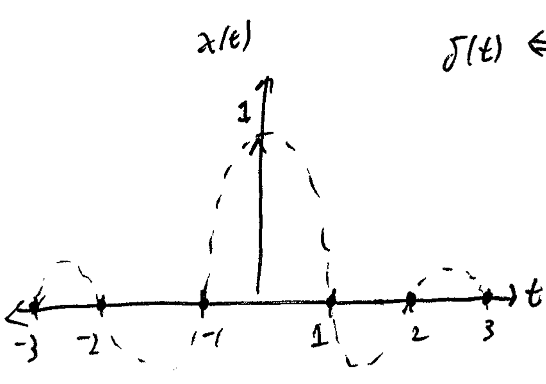
$= \sum_k 2 \, \text{rect}\left(\frac{\omega}{2\pi} - 4\pi k\right)$



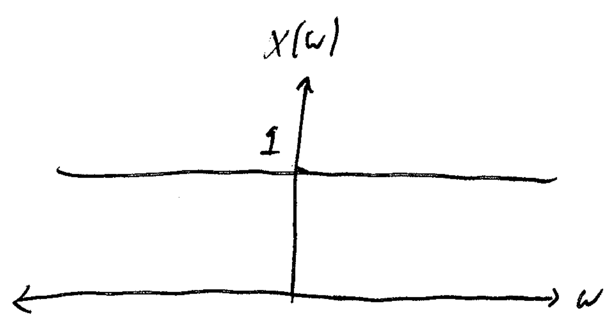


c: $x(t) = \text{sinc}(t) \sum_k \delta(t-k)$

$X(\omega) = \sum_k 1 \, \text{rect}\left(\frac{\omega}{2\pi} - 2\pi k\right)$

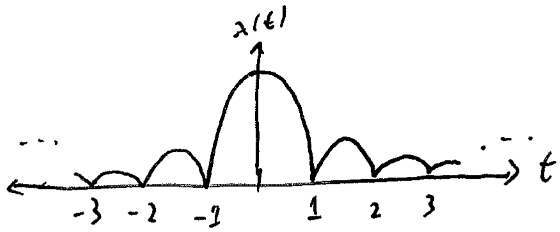


$\delta(t) \Longleftrightarrow 1$

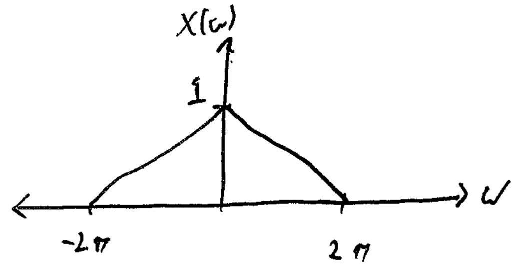


2) $y[n] = x(nT)$

a: $x(t) = (\text{sinc}(t))^2 \qquad T = \frac{3}{8}$

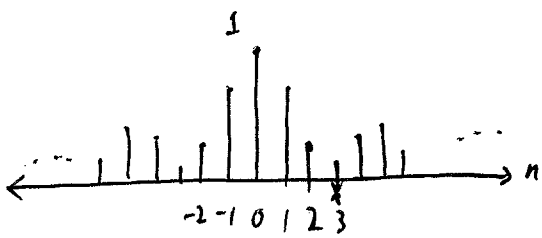


$$X(\omega) = \frac{1}{2\pi}\,\text{rect}\left(\frac{\omega}{2\pi}\right) * \text{rect}\left(\frac{\omega}{2\pi}\right)$$



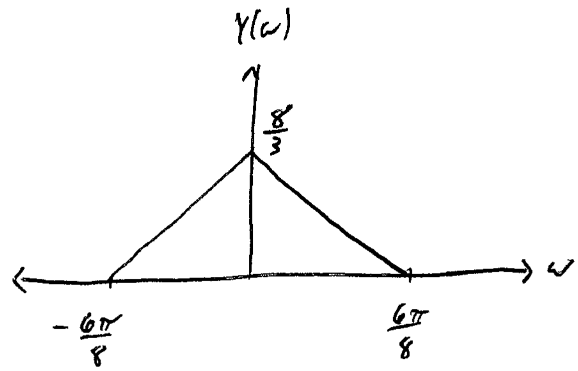
$$X(\omega) = \begin{cases} \frac{1}{2\pi}\omega + 1 & -2\pi < \omega < 0 \\ 1 - \frac{1}{2\pi}\omega & 0 < \omega < 2\pi \\ 0 & |\omega| > 2\pi \end{cases}$$

$$y[n] = (\text{sinc}(nT))^2$$
$$= \left(\text{sinc}\left(\frac{3}{8}n\right)\right)^2$$



Note: $\frac{\sin \omega_c n}{\pi n} \overset{DTFT}{\Longleftrightarrow} \begin{cases} 1 & |\omega| < \omega_c \\ 0 & ow \end{cases}$

$$Y(\omega) = \frac{1}{2\pi}\,\text{rect}\left(\frac{4}{3\pi}\omega\right)\frac{8}{3} * \frac{8}{3}\,\text{rect}\left(\frac{4}{3\pi}\omega\right)$$



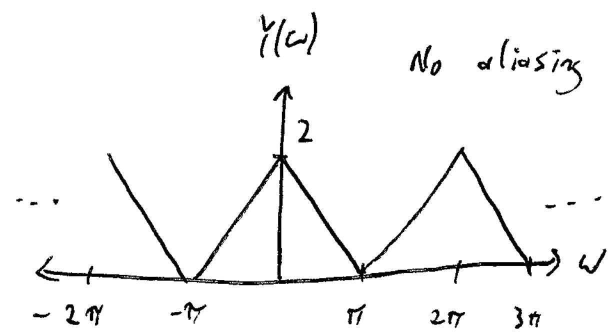
No aliasing $\left(\frac{6\pi}{8} < \pi\right)$

$$Y(\omega) = \begin{cases} \frac{8}{3}\left(1 + \frac{8}{6\pi}\omega\right) & -\frac{6\pi}{8} < \omega < 0 \\ \frac{8}{3}\left(1 - \frac{8}{6\pi}\omega\right) & 0 < \omega < \frac{6\pi}{8} \\ 0 & \pi > |\omega| > \frac{6\pi}{8} \end{cases}$$

b: $x(t) = (\text{sinc}(t))^2 \qquad T = \frac{1}{2}$

Plots of $x(t)$ & $X(\omega)$ are the same as part a:

$$y[n] = \left(\text{sinc}\left(\frac{1}{2}n\right)\right)^2$$



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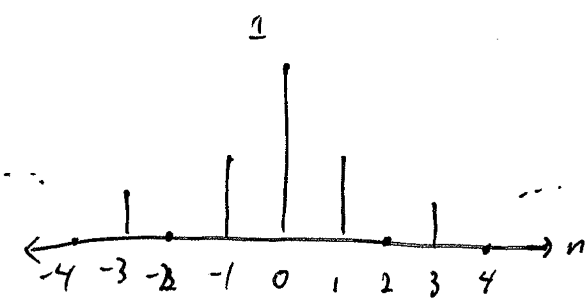
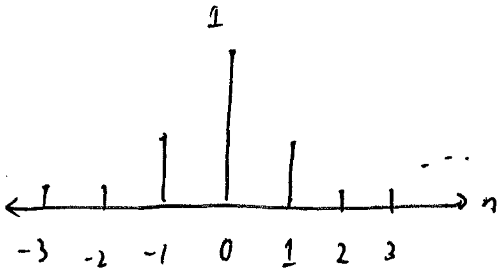


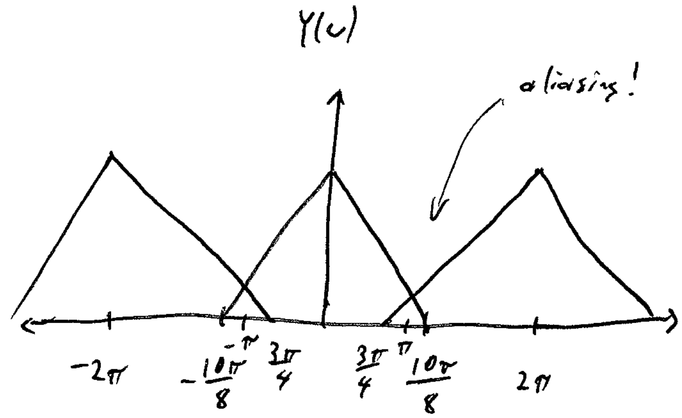
$$Y(\omega) = \begin{cases} 2\left(1 + \frac{1}{\pi}\omega\right) & -\pi < \omega < 0 \\ 2\left(1 - \frac{1}{\pi}\omega\right) & 0 < \omega < \pi \end{cases}$$

c: $x(t) = (\text{sinc}(t))^2 \qquad T = \frac{5}{8}$

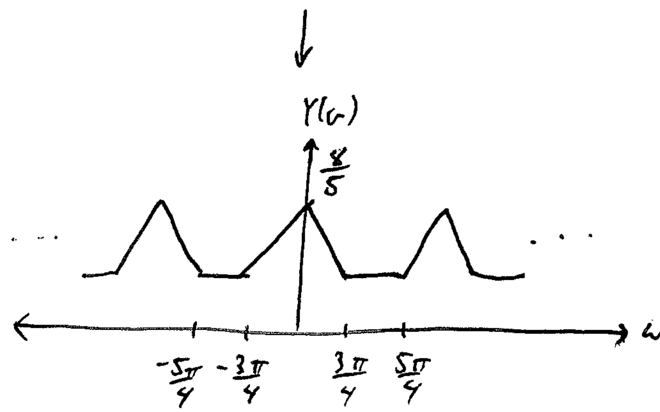
Plots of $x(t)$ & $X(\omega)$ are as in part a:

$y[n] = \left(\text{sinc}\left(\frac{5}{8}n\right)\right)^2$





$$Y(\omega) = \begin{cases} \frac{8}{5}\left(1 + \frac{4}{5\pi}\omega\right) & -\frac{3\pi}{4} < \omega < 0 \\ \frac{8}{5}\left(1 - \frac{4}{5\pi}\omega\right) & 0 < \omega < \frac{3\pi}{4} \\ \frac{16}{25} & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$



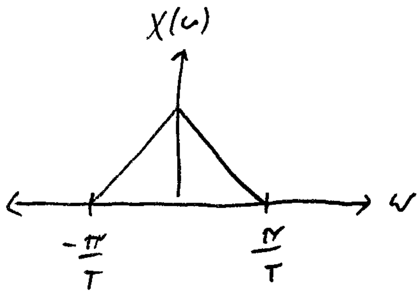
3)

$$y[n] = x(nT)$$

$$s(t) = \sum_k y[k]\,\delta(t-kT)$$

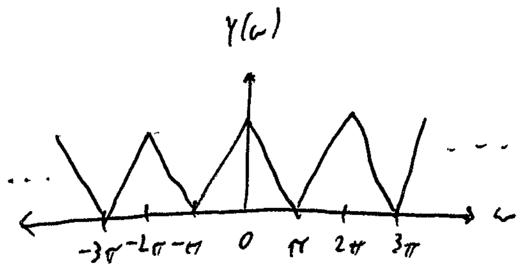
$$z(t) = s(t) * h(t)$$

a:



b: $Y(\omega) = \frac{1}{T} X\left(\frac{1}{T}\omega\right)$ for $|\omega| < \pi$

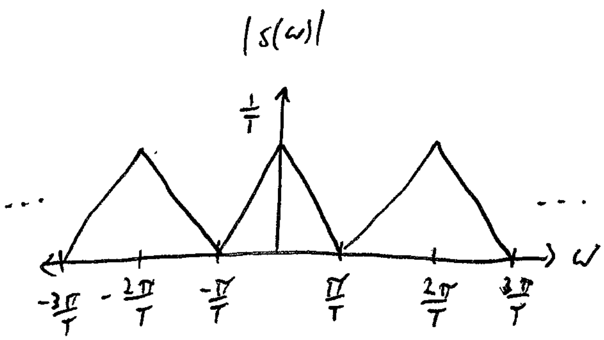
c:



d: $s(t) = \sum_k x(kT)\,\delta(t-kT) = \sum_k x(t)\,\delta(t-kT) = x(t) \sum_k \delta(t-kT)$

$$S(\omega) = \frac{1}{2\pi} \sum_k X\left(\omega - \frac{2\pi k}{T}\right) \frac{2\pi}{T} = \frac{1}{T} \sum_k X\left(\omega - \frac{2\pi k}{T}\right)$$

e:

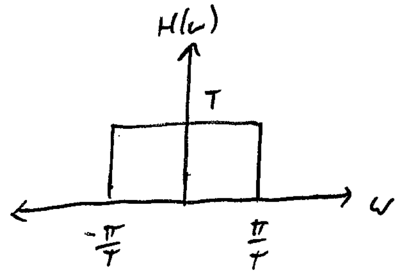


f: $Z(\omega) = S(\omega)\, H(\omega)$

$= \cancel{\left(\frac{?}{T} H(\omega) \sum_k X(\omega)\right)}$

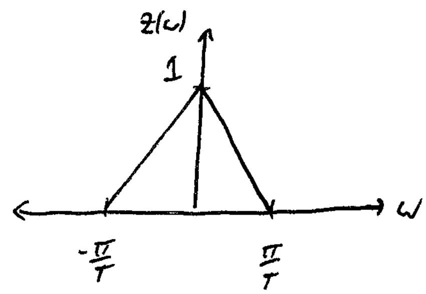
$= \frac{1}{T} H(\omega) \sum_k X\left(\omega - \frac{2\pi k}{T}\right)$

g: $H(\omega) = T\,\mathrm{rect}\left(\frac{\omega}{2\pi/T}\right)$



$Z(\omega) = X(\omega)$

h:



4) $y(t) = x(t) \sum_k \delta(t - kT)$ $\quad T = 1$

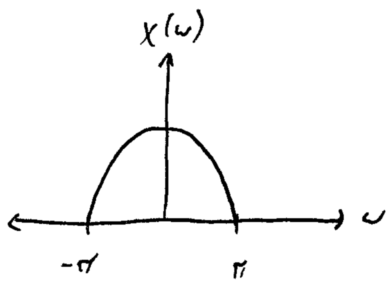
$x(t)$ band-limited $|\omega| < \pi$

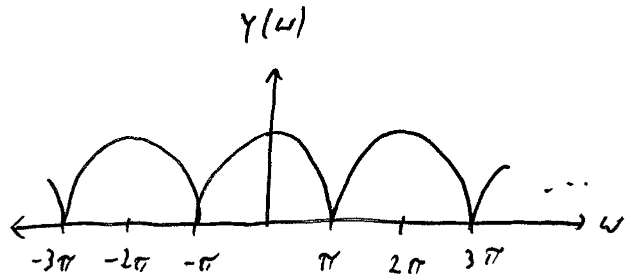
$z(t) = y(t) * h(t)$ $\quad h(t) = \text{sinc}(t)$

a: $Y(\omega) = \frac{1}{2\pi} X(\omega) * 2\pi \sum_k \delta(\omega - 2\pi k)$

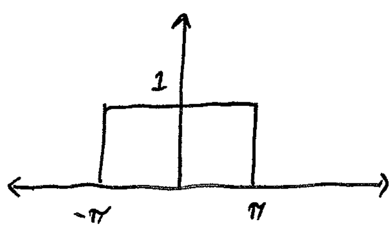
$= \sum_k X(\omega - 2\pi k)$

b:



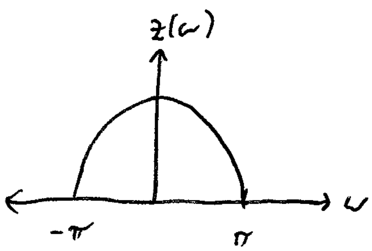


c: $h(t) = \text{sinc}(t) \Rightarrow H(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$



d: $Z(\omega) = X(\omega)$

e:



f: $Z(\omega) = X(\omega) \Rightarrow z(t) = x(t)$