EE301 Homework #9: The DFT and DTFT

Problem 1 - *DFT*
For each of the following discrete-time signals $x(n)$, calculate the DFT X_k for $0 \leq k < N$. In each case, assume that m is an integer.

(a) $x(n) = \delta(n)$ for $0 \leq n < N$.

(b) $x(n) = \delta(n-m)$ for $0 \leq n, m < N$.

(c) $x(n) = e^{j\frac{2\pi nm}{N}}$ for $0 \leq n, m < N$.

(d) $x(n) = \cos\left(\frac{2\pi nm}{N}\right)$ for $0 \leq n, m < N$.

(e) $x(n) = \sin\left(\frac{2\pi nm}{N}\right)$ for $0 \leq n, m < N$.

Problem 2 - *DFT of a Sine Wave*
Let $x(n) = e^{j\omega n}$ for $0 \leq n < N$ and let X_k be its DFT.

(a) Calculate an explicit expresion for X_k that is correct for any value of ω.

(b) Sketch a plot of $|X_k|$ for $\omega = \pi/N$ and $N = 20$.

(c) Calculate a simplified expression for X_k when $\omega = 2\pi m/N$ where m is an integer

(d) Sketch a plot of $|X_k|$ for $\omega = 2\pi/N$ and $N = 20$.

Problem 3 - *Parseval's Theorem for the DFT*

(a) Let the functions $\phi_k(n)$ for $0 \leq n, k < N$ have the property that

$$< \phi_k, \phi_l >= \alpha\delta(k-l)$$

and let

$$x(n) = \sum_{k=0}^{N-1} X_k \phi_k(n)$$

then prove that

$$\sum_{n=0}^{N-1} |x(n)|^2 = \alpha \sum_{k=0}^{N-1} |X_k|^2$$

(b) Specify the functions $\phi_k(n)$, the constant α, and the form of the innerproduct $< \phi_k, \phi_l >$ so that the transform described in part a) is a DFT as described in lecture.

Problem 4 *DTFT Transforms*
Compute the DTFT, $X(\omega)$, for the following signals.

(a) $x(n) = u(n) - u(n-m)$ for $m \geq 0$.

(b) $x(n) = \delta(n-m)$ for m an integer.

(c) $x(n) = e^{(j\omega_0 - a)n}u(n)$

(d) $x(n) = \cos(\omega_0 n + \phi)$

(e) $x(n) = \sin(\omega_0 n + \phi)$

(f) $x(n) = a^n u(n)$ where $|a| < 1$

(g) $x(n) = a^{|n|}$ where $|a| < 1$

(h) $x(n) = na^n u(n)$ where $|a| < 1$

(i) $x(n) = a^{n-1}u(n-1)$ where $|a| < 1$

Problem 5 *Difference Equations*
Consider the discrete time system $y(n) = T[x(n)]$ with input $x(n)$ and output $y(n)$ which obeys the following difference equation

$$y(n) = 2r\cos(\theta)y(n-1) - r^2 y(n-2) + x(n)$$

where $|r| < 1$ and θ are real valued constants.

(a) Prove the system $T[\cdot]$ is linear.

(b) Prove the system $T[\cdot]$ is time invariant.

(c) Calculate the frequency response $H(\omega)$ of the system.

(d) Calculate the impulse response $h(n)$ of the system.