EE301 Homework #8: System Analysis with the CTFT

Problem 1 - *Convolution and CTFT's*
For each of the following, calculate $X(\omega)$, $Y(\omega)$, $Z(\omega) = X(\omega)Y(\omega)$, and $z(t)$.

(a) $x(t) = e^{-t}u(t)$ and $y(t) = e^{-t}u(t)$

(b) $x(t) = (e^{-t}u(t)) * (e^{-t}u(t))$ and $y(t) = e^{-t}u(t)$

(c) $x(t) = \frac{t^{n-1}}{(n-1)!}e^{-t}u(t)$ and $y(t) = e^{-t}u(t)$

(d) $x(t) = e^{-t}u(t)$ and $y(t) = e^{-2t}u(t)$

(e) $x(t) = e^{-t}u(t)$ and $y(t) = te^{-2t}u(t)$

Problem 2 - *Frequency analysis of LTI systems*
Consider a LTI system with frequency response $H(\omega)$, input $x(t)$, and output $y(t)$.

(a) Derive an expression for
$$\int_{-\infty}^{\infty} h(t)dt$$
in terms of the function $H(\omega)$.

(b) Derive an expression for $h(0)$ in terms of $H(\omega)$.

(c) If the input is $x(t) = a$, then express the output $y(t)$ in terms of a and $H(\omega)$.

(d) If the input is $x(t) = a$, then express the output $y(t)$ in terms of a and $h(t)$.

(e) You are asked to design a LTI system with a DC gain of A. What do you know about the impulse response of the system?

(f) You are asked to design a LTI system with a DC gain of A. What do you know about the frequency response of the system?

Problem 3 *Transfer functions for LTI systems.*
For an LTI system T we have
$$T[e^{-2t}u(t)] = te^{-t}u(t) + 2e^{-2t}u(t)$$
Determine the transfer function, $H(\omega) = \frac{Y(\omega)}{X(\omega)}$, for this system.

Problem 4 *Frequency analysis of linear differential equations*
Consider the system with input $x(t)$ and output $y(t)$ described by the differential equation
$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} + x(t)$$
where the system is assumed to be initially at rest.

a) Calculate the frequency response of the system $H(\omega)$.

b) Express $H(\omega)$ as the ratio of factored polynomials.

Problem 5 - *Frequency analysis of linear differential equations*
Consider the system with input $x(t)$ and output $y(t)$ described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 10y(t) = \frac{dx(t)}{dt} - x(t)$$

where the system is assumed to be initially at rest.

(a) Determine the frequency response of the system $H(\omega)$.

(b) Determine the impulse response of the system $h(t)$.

(c) If the input to the system is $x(t) = e^{-t}u(t)$ find the corresponding output.

Problem 6 - *Inverse CTFT's*
Calculate the inverse CTFT's of the following transforms.

(a) $X(\omega) = \frac{1}{j\omega+5}$

(b) $X(\omega) = \frac{1}{(j\omega+5)^2}$

(c) $X(\omega) = \frac{1}{(j\omega+5)(j\omega+2)}$