

EE301 Homework #7: The Continuous-Time Fourier Transform

Problem 1 - Evaluating CTFTs.

Calculate the continuous-time Fourier transform for the following signals:

- a) $x(t) = e^{-at}u(t)$ for $a > 0$
- b) $x(t) = te^{-at}u(t)$ for $a > 0$
- c) $x(t) = \text{rect}(t)$
- d) $x(t) = \text{rect}\left(\frac{t-a}{b}\right)$ for any two real numbers a and b .
- e) $x(t) = \delta(t)$
- f) $x(t) = a\delta(t - b)$ for any two real numbers a and b .

Problem 2 - Evaluating inverse CTFTs.

Calculate the **inverse** CTFT for the following signals.

- a) $X(\omega) = \delta(\omega)$
- b) $X(\omega) = \delta(\omega - \omega_0)$
- c) $X(\omega) = \text{rect}(\omega)$

Problem 3 - Evaluating CTFTs.

Use the CTFT properties to compute the CTFT's of the following signals.

- a) $x(t) = \text{sinc}(t)$.
- b) $x(t) = \text{sinc}\left(\frac{t-a}{b}\right)$ for any two real numbers a and b .
- c) $x(t) = 1$
- d) $x(t) = e^{j\omega_0 t}$
- e) $x(t) = \cos(\omega_0 t)$
- f) $x(t) = \sin(\omega_0 t)$

Problem 4 - Properties of CTFTs.

For the following problems, let $X(\omega)$ and $Y(\omega)$ be the CTFT's of $x(t)$ and $y(t)$, respectively. Calculate the CTFT of each function in terms of the functions $x(t)$, $y(t)$, $X(\omega)$, and $Y(\omega)$.

- (a) $5x(t - a)$
- (b) $X(t)$
- (c) $x(t) * y(t)$
- (d) $x(t)y(t)$
- (e) $x(-t)$
- (f) $x(t)e^{j\omega_0 t}$
- (g) $\frac{1}{|a|}X\left(\frac{\omega}{a}\right)$

Problem 5 - Deriving CTFT Properties

Derive each of the following CTFT properties. Assume that in each case the CTFT of $x(t)$ and $y(t)$ are $X(\omega)$ and $Y(\omega)$ respectively.

- b) $x(-t) \stackrel{CTFT}{\Leftrightarrow} X(-\omega)$
- c) $x(t - t_0) \stackrel{CTFT}{\Leftrightarrow} X(\omega)e^{-j\omega t_0}$
- d) $x(at) \stackrel{CTFT}{\Leftrightarrow} \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$
- e) $X(\omega) = X^*(-\omega)$ if $x(t)$ is real
- h) $x(t)y(t) \stackrel{CTFT}{\Leftrightarrow} \frac{1}{2\pi}X(\omega) * Y(\omega)$
- j) $\frac{dx(t)}{dt} \stackrel{CTFT}{\Leftrightarrow} j\omega X(\omega)$

Problem 6 - Symmetry properties of the CTFT

For each of the following transforms, determine whether the corresponding time-domain signal is (i) real, purely imaginary, or complex, and (ii) even, odd, or neither even nor odd. Do this without evaluating the inverse CTFT.

- (a) $X(\omega) = \sin(2\omega) \cos(3\omega)$
- (b) $X(\omega) = \sin(\omega) e^{j(2\omega + \pi/2)}$
- (c) $X(\omega) = u(\omega) - u(\omega - 4\pi)$

Problem 7 - Duality property of the CTFT

Use the duality property to determine the CTFT of the following signals

- (a) $x(t) = \frac{1}{5 + j2\pi t}$
- (b) $x(t) = \frac{t}{(1+t^2)^2}$ (Hint: see question 4.12 in the textbook.)