EE301 Homework #6: Fourier Series Expansion

Problem 1 *Determining Fourier series coefficients.*
Each of the following functions is periodic with period T. For each function sketch the real and imaginary parts of the function on the interval $[0, 2T]$ and calculate the Fourier series coeffcients.

(a) $x(t) = e^{j2\pi t/3}$ with period $T = 3$.

(b) $x(t) = \sin(2\pi t/3) + 3\cos(\pi t/6)$ with period $T = 12$.

(c) $x(t) = \text{rect}(t)$ for $|t| < T/2$ with period $T = 2$. (put in simplest form)

(d) $x(t) = \Lambda(t)$ for $|t| < T/2$ with period $T = 2$. (put in simplest form)

Problem 2 *Properties of Fourier series.*
Suppose that the Fourier series coefficients for the function $x(t)$ with period T are given as a_k, and the Fourier series coefficients for the function $y(t)$ with period T are given as b_k. Prove the following relationships.

(a) If $y(t) = \frac{d\,x(t)}{dt}$ then $b_k = jk\frac{2\pi}{T}a_k$.

(b) If $y(t) = x(-t)$ then $b_k = a_{-k}$.

(c) If $x(t)$ is real, then $a_k = a^*_{-k}$.

(d) If $x(t)$ is real and $x(t) = x(-t)$, then a_k are real and $a_k = a_{-k}$.

Problem 3 *Reconstructing signals from Fourier series coefficients.*
In each of the following, the Fourier series coefficients and the period of a signal are specified. Determine the signal $x(t)$ in each case.

(a) $a_k = (\frac{1}{2})^{|k|}$ and $T = 2$.

(b) $a_k = \begin{cases} jk & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$ and $T = 4$.

(c) $a_k = \cos(\pi k/4)$ and $T = 4$.

Problem 4 *Fourier series and LTI systems.*
Suppose that the signal $x(t)$ is periodic with period T and Fourier series coefficients a_k. Let $y(t) = h(t) * x(t)$ where $h(t)$ is the impulse response of an LTI system.

(a) Show that $y(t)$ is also periodic with period T.

(b) Show that the Fourier series coefficients of $y(t)$ have the form $b_k = c_k\,a_k$ where c_k are multiplicative constants.

(c) Derive an expression for the multiplicative constants c_k.