EE301 Homework #3: DT and CT Impulse Response of LTI Systems

Problem 1 - *Determining the impulse response of DT LTI systems.*
For each of the following systems with input x and output y, i) prove that the system is linear; ii) prove that the system is time invariant; iii) compute the system's impulse response. Simplify your answer as much as possible.

(a) $y_n = \sum_{k=0}^{\infty} b_k x_{n-k}$

(b) $y_n = \frac{1}{3}\left(x_n - \frac{1}{2}(x_{n-1} + x_{n+1})\right)$

(c) $y_n = \frac{1}{2}y_{n-1} + x_n$

Problem 2 *Discrete-time Impulse Response*
Consider the discrete-time LTI system described by the equation

$$y_n = x_n - 3x_{n-1} + 2x_{n-2}$$

(a) Compute the impulse response of the system.

(b) Express the system in the form $y_n = x_n * h_n$.

(c) Find the output when the input is given by $x_n = u_n$.

(d) Find the output when the input is given by $x_n = 1$.

Problem 3 - *Determining the impulse response of CT LTI systems.*
For each of the following systems with input x and output y, i) prove that the system is linear; ii) prove that the system is time invariant; iii) compute the system's impulse response. Simplify your answer as much as possible.

(a) $y(t) = \int_{-\infty}^{\infty} r(\tau - t)x(\tau)d\tau$

(b) $y(t) = x(t) + 2x(t+1) + 3x(t-1)$

(c) $\frac{dy(t)}{dt} = -x(t)$

Problem 4 - *Discrete-time Impulse Response for LTI systems.*
A time invariant system $T[\cdot]$ is observed to have the following input/output relationships.

$$\begin{aligned} \delta_{n-1} + 2\delta_{n-2} &= T\left[\,\delta_n + 2\delta_{n-2}\,\right] \\ \delta_{n-1} + 2\delta_{n-3} &= T\left[\,3\delta_{n-2}\,\right] \\ \delta_{n+1} + 2\delta_n + \delta_{n-1} &= T\left[\,\delta_{n-3}\,\right] \end{aligned}$$

(a) Prove that the system is linear or nonlinear.

(b) Compute the response to an input of δ_n, that is compute $T[\delta_n]$.

Problem 5 - *CT Convolution.*
Find the outputs of the following LTI systems with the following inputs.

(a) Impulse response of $h(t) = u(t+1) - u(t-1)$; input of $x(t) = u(t) - u(t-2)$

(b) Impulse response of $h(t) = e^{-at}u(t)$; input of $x(t) = u(t)$ for $a > 0$.

(c) Impulse response of $h(t) = e^{-at}\cos(\omega t)u(t)$; input of $x(t) = u(t)$ for $a > 0$ and $\omega \in \Re$.

(d) Impulse response of $h(t) = e^{-at}u(t)$; input of $x(t) = e^{-bt}u(t)$ for $a \neq b > 0$.

Problem 6 - *DT Convolution.*
Calculate the output of a LTI system with impulse response $h(n)$, input $x(n)$, and output $y(n)$.

(a) $h(n) = a^n u(n)$ and $x(n) = b^n u(n)$ where $a \neq b$.

(b) $h(n) = a^n u(n)$ and $x(n) = a^n u(n)$

(c) $h(n) = a^n u(n)$ and $x(n) = \cos(\omega n)$ where $|a| < 1$.

(d) $h(n) = u(n) - u(n-N)$ and $x(n) = u(n) - u(n-P)$ for $P > N$.