EE301 Homework #2: System Properties

Problem 1 - *Formal Logic Operations.* Use a truth table to prove that the following logical relationships are equivalent.

(a) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

(b) $(\neg(P \Rightarrow Q)) \Leftrightarrow (\neg Q \& P)$

(c) $\neg(P \vee Q) \Leftrightarrow (\neg P \& \neg Q)$

(d) $\neg(P \& Q) \Leftrightarrow (\neg P \vee \neg Q)$

Problem 2 - *Formal Logic Qualifiers.* For each of the following logical expressions, do the following: i) negate the expression, ii) give a real world example for the sets (i.e. A and B) and the logical operators (i.e. P and Q), and iii) write an english sentence using the examples of part ii that expresses the concepts of the expression.

(a) $\forall x \in A, Px$

(b) $\forall x \in A, Px \Rightarrow Qx$

(c) $\exists\, x \in A\ s.t. \neg Px\ \vee Qx$

(d) $\forall x \in A\ \exists\, y \in B\ s.t.\ Pxy$

(e) $\exists\, x \in A\ s.t. \forall y \in B, Pxy$

Problem 3 - *Negation of a Logical Statement.* Consider the following definition: "A signal $x(t)$ is **bounded** if there exists an $M > 0$ such that for all t, $|x(t)| < M$."

(a) Write a formal logical expression for the definition of a bounded signal. Specify all sets and logital operators.

(b) Negate the expression of part (a) to derive a logical expression for the definition of an **unbounded** signal (i.e. a signal that is not bounded).

(c) Use the result of part (b) to write an English definition for unbounded signals.

(d) Prove that the signal $x(t) = \sin(t)$ is bounded on $\Re$.

(e) Prove that the signal $x(t) = |t|$ is unbounded on $\Re$.

Problem 4 - *CT System Properties.* Consider a CT system with input $x(t)$ and output $y(t)$. For each of the following systems, i) prove that it is linear or give a counter example, ii) prove that it is time-invariant or give a counter example, iii) determine whether it is causal or noncausal, and iv) determine if it is a memoryless or memory system.

(a) $y(t) = u(t)x(t)$

(b) $y(t) = x(\sin(t))$

(c) $y(t) = \sin(x(t))$

(d) $y(t) = \frac{dx(t)}{dt}$

(e) $y(t) = x(2t) - x(t-1)$

(f) $y(t) = x(0)$

(g) $y(t) = \int_0^t x(\tau)d\tau$

Problem 5 - *DT System Properties.* Consider a system with input $x[n]$ and output $y[n]$. For each of the following systems, i) prove that it is linear or give a counter example, ii) prove that it is time-invariant or give a counter example.

(a) $y[n] = x[n] + 1$

(b) $y[n] = x[2n]$ (This operation is known as *decimation*.)

(c) $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

(d) $y[n] = \begin{cases} x[n], & x[n] < 4 \\ 4, & \text{else} \end{cases}$