

EE301 Homework #11: Z-Transforms

Problem 1 - Computing the Z-transforms

For each of the following functions, compute the Z-transform along with a region-of-convergence (ROC).

- a) $x(n) = a^n u(n)$
- b) $x(n) = a^n u(-n)$
- c) $x(n) = a^{|n|}$ for $a < 1$

Problem 2 - Computing the inverse Z-transforms

For each of the following parts, determine the inverse of the Z-transform $\frac{1}{az^{-1} + 1}$ and specify the associated ROC. Assume that $a \in \mathfrak{R}$.

- a) Determine the inverse that is right-sided.
- b) Determine the inverse that is left-sided.
- c) Determine the inverse that is stable.

Problem 3 - Poles and Zeros of Z-transforms

Consider the Z-transform

$$H(z) = \frac{1}{(2z^{-1} - 1)((1/2)z^{-1} - 1)}$$

- a) Find the poles and zeros of the Z-transform.
- b) Sketch the poles, zeros and the possible ROCs
- c) For each ROC, determine if the impulse response is causal, right sided, or left sided.
- d) For each ROC, determine if the impulse response is stable or unstable.
- e) For each ROC, compute the corresponding signal $h(n)$.

Problem 4 - Z-transforms for transfer functions

Consider the following difference equation

$$y(n) = ay(n - 1) + x(n)$$

- a) Calculate the transfer function $H(z) = \frac{Y(z)}{X(z)}$, and find its poles and zeros.

- b) Calculate the impulse response $h(n)$ using an ROC of $|z| > a$. For what values of a is the system stable?
- c) Calculate the impulse response $h(n)$ using an ROC of $|z| < a$. For what values of a is the system stable?