EE301 Homework #10: Sampling and Reconstruction

Problem 1 - *Computing CTFT Transforms*
For each of the following functions, compute the CTFT then sketch the function $x(t)$ and its Fourier transform $X(\omega)$. (Hint: Use CTFT property 12 from notes.)

a) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k/2)$

b) $x(t) = \text{sinc}(t) \sum_{k=-\infty}^{\infty} \delta(t - k/2)$

c) $x(t) = \text{sinc}(t) \sum_{k=-\infty}^{\infty} \delta(t - k)$

Problem 2 - *Sampling and DTFT's*
Consider the functions

$$y(n) = x(nT)$$

For each example, i) sketch $x(t)$, ii) calculate $X(\omega)$ the CTFT of $x(t)$, iii) sketch $|X(\omega)|$, iv) sketch $y(n)$, v) calculate $Y(\omega)$ the DTFT of $y(n)$, vi) sketch $|Y(\omega)|$, vii) indicate if there is aliasing.

(a) $x(t) = (\text{sinc}(t))^2$ and $T = 3/8$.

(b) $x(t) = (\text{sinc}(t))^2$ and $T = 1/2$.

(c) $x(t) = (\text{sinc}(t))^2$ and $T = 5/8$.

Problem 3 - *Sampling and Reconstruction*
A signal $x(t)$ is sampled at period T to form $y(n)$.

$$y(n) = x(nT)$$

The signal $y(n)$ is then used as the input to an impulse generator to form $s(t)$.

$$s(t) = \sum_{k=-\infty}^{\infty} y(n)\delta(t - kT)$$

The signal $s(t)$ is then filtered to form the final output $z(t)$ using the filter $H(\omega)$.

(a) Sketch a general function $|X(\omega)|$ which is bandlimited to $|\omega| < \frac{\pi}{T}$.

(b) Calculate $Y(\omega)$ in terms of $X(\omega)$.

(c) Sketch $|Y(\omega)|$ for a typical function $X(\omega)$.

(d) Calculate $S(\omega)$ in terms of $X(\omega)$.

(e) Sketch $|S(\omega)|$.

(f) Calculate $Z(\omega)$ in terms of $X(\omega)$.

(g) Calculate $Z(\omega)$ in terms of $X(\omega)$ assuming that $H(\omega) = T\text{rect}\,(T\omega/(2\pi))$

(h) Sketch $|Z(\omega)|$ assuming that $H(\omega) = T\text{rect}\,(T\omega/(2\pi))$

Problem 4 - *Sampling and reconstruction*
Consider a sampling system

$$y(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

where $T = 1$ and $x(t)$ is a function that is band-limited to $|\omega| < \pi$. Then, consider the signal

$$z(t) = y(t) * h(t)$$

where $h(t) = \text{sinc}(t)$.

(a) Determine $Y(\omega)$ in terms of $X(\omega)$.

(b) Sketch $Y(\omega)$ for a typical function $X(\omega)$.

(c) Determine and sketch $H(\omega)$.

(d) Determine $Z(\omega)$ in terms of $X(\omega)$.

(e) Sketch $Z(\omega)$ for a typical function $X(\omega)$.

(f) Determine $z(t)$ in terms of $x(t)$.