EE301 Homework #1: Signal Properties

Problem 1 - *Complex numbers.*

(a) Express the following complex numbers in the $polar$ form $z = Ae^{j\theta}$.

i. $z = 1 + j\sqrt{3}$

ii. $z = a + jb$ for $a > 0$

iii. $z = (a + jb)^3$ for $a > 0$

iv. $z = \frac{e^{j\psi}}{e^{j\phi}}$

v. $z = \frac{e^{j\pi/3}-1}{1+j\sqrt{3}}$

(b) Express the following complex numbers in $rectangular$ form $z = a + jb$.

i. $z = e^{j\theta}$

ii. $z = e^{a+j\theta}$

iii. $z = \frac{e^{j\psi}}{e^{j\phi}}$

iv. $z = \frac{1}{c+jd}$

v. $z = \frac{e+jf}{c+jd}$

Problem 2 - *Rectangular to polar conversion.*

(a) Let $Ae^{j\theta} = a + jb$ for $a^2 + b^2 > 0$. Precisely specify a function $\theta = f(a, b)$ which is correct for all values of a and b and results in a positive value of A.

Problem 3 *Transformations of independent variable.* Let $x(t) = \sin(2\pi t)u(t + 1/2)u(-t + 1/2)$. Sketch and label carefully the following signals.

(a) $x(t)$

(b) $x(t/2)$

(c) $x(-t - 1/2)$

Problem 4 - *Signal Property: Fundamental periods.*

(a) For each of the following continuous-time signals, determine if the signals are periodic, and specify their fundamental period.

i. $x(t) = b\cos(2\pi ft + \theta)$

ii. $x(t) = b\cos(\omega t + \theta)$

iii. $x(t) = b\cos(\omega_1 t + \theta) + c\sin(\omega_2 t + \phi)$ where $\omega_2 = 2\omega_1$

iv. $x(t) = b\cos(\omega_1 t + \theta) + c\sin(\omega_2 t + \phi)$ where $\omega_2 = \sqrt{2}\omega_1$

(b) For each of the following discrete-time signals, determine if the signals are periodic, and specify their fundamental period.

i. $x(k) = b\cos(2\pi f k + \theta)$ where $f = \frac{1}{2}$.
ii. $x(k) = b\cos(2\pi f k + \theta)$ where $f = \frac{1}{8}$.
iii. $x(k) = b\cos(2\pi f k + \theta)$ where $f = \frac{2}{8}$.
iv. $x(k) = b\cos(2\pi f k + \theta)$ where $f = \frac{7}{8}$.
v. $x(k) = b\cos(2\pi f k + \theta)$ where $f = \frac{6}{11}$.

Problem 5 - *Signal Property: Energy and Power.* Calculate the energy and power for the following signals.

(a) $x(t) = e^{-t}u(t)$

(b) $x(t) = \cos(t)$

(c) $x[n] = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$

(d) $x[n] = \cos\left(\frac{\pi}{4}n\right)$

Problem 6 - *Discrete-Time Impulse and Step Functions.*

(a) Calculate $\sum_{n=-\infty}^{\infty} n^2\delta(n-3)$

(b) Sketch the function $x(n) = n^2\delta(n-T)$ for $T = 0, 1, \cdots, 4$.

(c) Show that $u(n) = \sum_{k=-\infty}^{\infty} u(k)\delta(n-k)$

(d) Show that for all functions $x(n)$, $x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$