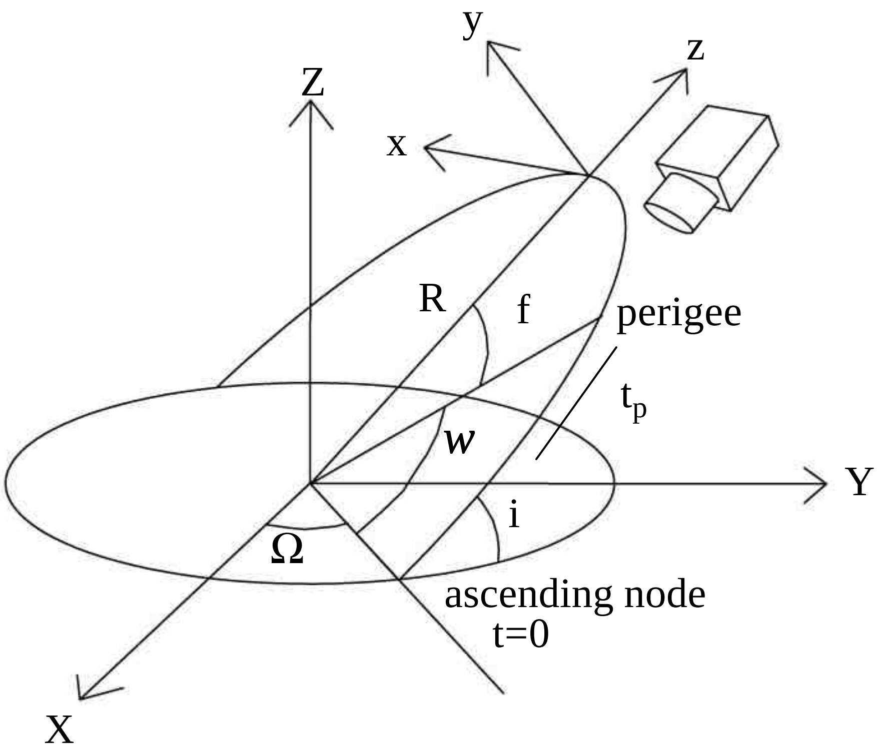
Development of the Condition Equations for a Space Based Pushbroom Camera (Using SPOT as an Example)

Development of SPOT Condition Equation – Good Model for Generic Pushbroom Camera from LEO



Must have approximations for

$\Omega, \mathrm{i}, w, \mathrm{a}, \mathrm{e}$

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

t_f: time at frame center, in header (metadata)

delta-t: delta-time from frame center, equals 0.001504 sec * line,

$$t = t_f + \Delta t$$

orbit period, $t = 2p\sqrt{\dfrac{a^3}{GMe}}$

GMe = 398600.5E09 m^3/s^2

$a_s = r_e + alt_s$,6378137m + 822000m = 7200137m

$$t = 2p\sqrt{\frac{7200137^3}{398600.5E09}}$$

$$t = 6080.259 \text{ min}$$

$$t = 101.338 \text{ sec}$$

t_p: time from ascending node to perigee

Condition Equation cont'd.

$$t_p = \frac{t}{p}\tan^{-1}\left[\frac{\sqrt{1-e}}{\sqrt{1+e}}\tan(w/2)\right] - \frac{t}{2p}\frac{e\sqrt{1-e^2}\sin w}{1+e\cos w}$$

f: true anomaly

$$\sin f = \frac{\sqrt{1-e^2}\sin E}{1-e\cos E}$$

$$\Delta t_p = t - t_p$$

$$\cos f = \frac{\cos E - e}{1-e\cos E}$$

$$M_n = \frac{2p\Delta t_p}{t}, \text{ mean anomoly}$$

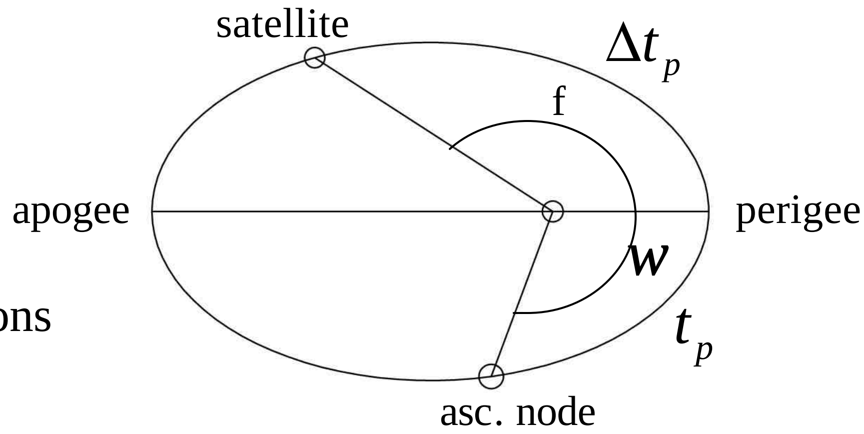
$$f = \tan^{-1}\left(\frac{\sin f}{\cos f}\right)$$

$$E = e\sin E + M_n, \text{ (kepler equation, E : eccentric anomaly)}$$

solve iteratively for E

$$R_s = a(1-e\cos E) \text{ ; vector from earth center to satellite}$$

$$\begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix} = \mathbf{M_b}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Construct $\mathbf{M_b}$ from 3 sequential rotations applied to XYZ (ECEF) to bring them parallel to xyz (instantaneous satellite system)

Condition Equation, cont'd.

3 rotations needed to construct M

first $M_Z(a_1)$

second $M_X(a_2)$

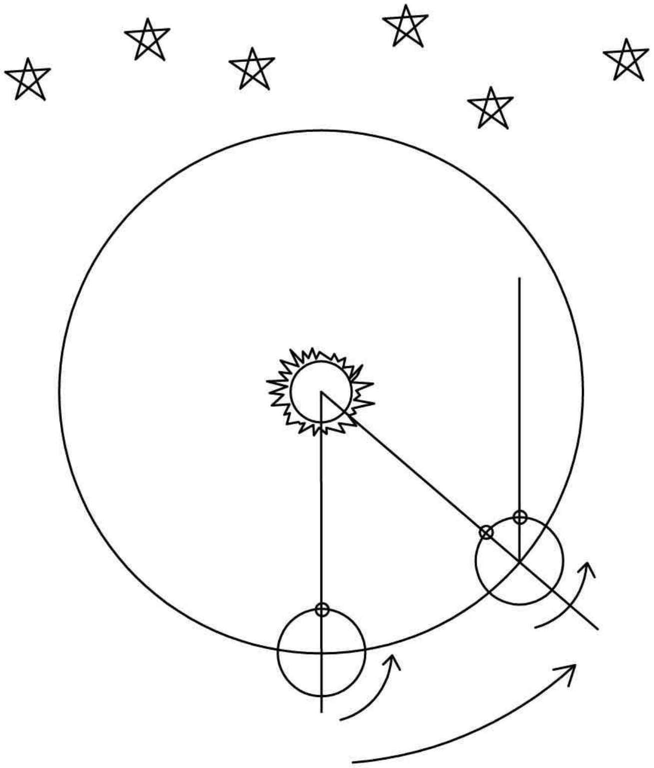
third $M_Y(a_3)$

$a_1 = \Omega - w_e t$

t : time since ascending node

Ω : longitude of asc. node at satellite passage of the asc. node

w_e : rotation rate of the earth (small confusion here : we want rotation corresponding to sidereal day, but measured in time units based on solar day)



When we make one rotation with respect to the sun (solar day) we have made more than one rotation with respect to the stars (sidereal day). i.e. solar day is longer than sidereal day. In fact, in one year there are 365.25 solar days, and 366.25 sidereal days (one more)

Condition Equation, cont'd.

Earth rotation rate (solar rate) is 2π radians per 24 hours, or 0.00007272205 rad/sec

Earth rotation rate (sidereal rate) is faster by factor of (366.25/365.25), or 0.00007292115 rad/sec

From the point of view of an earth observing camera in orbit, the earth motion will be at the sidereal rate

The first rotation, about Z, puts X' through the ascending node.

$$\mathbf{M_1} = \begin{bmatrix} \cos a_1 & \sin a_1 & 0 \\ -\sin a_1 & \cos a_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The second rotation, about X', puts Y'' up into the orbit plane (i), then another 90 degrees, so that it is normal to the orbit plane

$$a_2 = i + 90^\circ \qquad \mathbf{M_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a_2 & \sin a_2 \\ 0 & -\sin a_2 & \cos a_2 \end{bmatrix}$$

Condition Equation, cont'd.

The third rotation is about Y'' and moves X''' through omega, f, and another 90 degrees so it is pointing in the instantaneous direction of motion of the satellite (tangent to the orbit)

$$a_3 = w + f + 90^\circ$$

$$\mathbf{M_3} = \begin{bmatrix} \cos a_3 & 0 & -\sin a_3 \\ 0 & 1 & 0 \\ \sin a_3 & 0 & \cos a_3 \end{bmatrix}$$

The composite rotation $\mathbf{M_b}$ is the product of these three elementary rotations

$$\mathbf{M_b} = \mathbf{M_3 M_2 M_1}$$

With the following relationships,

$$\mathbf{M_b} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix}; \qquad \mathbf{M_b^T} \begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Condition Equation, cont'd.

The XYZ obtained in this way will be only approximately correct and we must allow for refinements, modeled as second order polynomials of time:

$$\Delta X = dX_0 + dX_1 \Delta t + dX_2 \Delta t^2$$

$$\Delta Y = dY_0 + dY_1 \Delta t + dY_2 \Delta t^2$$

$$\Delta Z = dZ_0 + dZ_1 \Delta t + dZ_2 \Delta t^2$$

Likewise the attitude (orientation) produced by the prior rotation matrix will be only approximately correct and we must allow for refinements to the attitude, again modeled as second order polynomials of time:

$$\Delta w = dw_0 + dw_1 \Delta t + dw_2 \Delta t^2$$

$$\Delta j = dj_0 + dj_1 \Delta t + dj_2 \Delta t^2$$

$$\Delta k = dk_0 + dk_1 \Delta t + dk_2 \Delta t^2$$

Condition Equation, cont'd.

We put these small refinement rotations into matrix as follows:

$$\mathbf{M}_{\mathbf{a}} = \mathbf{M}_{\Delta k}\mathbf{M}_{\Delta j}\mathbf{M}_{\Delta w}$$

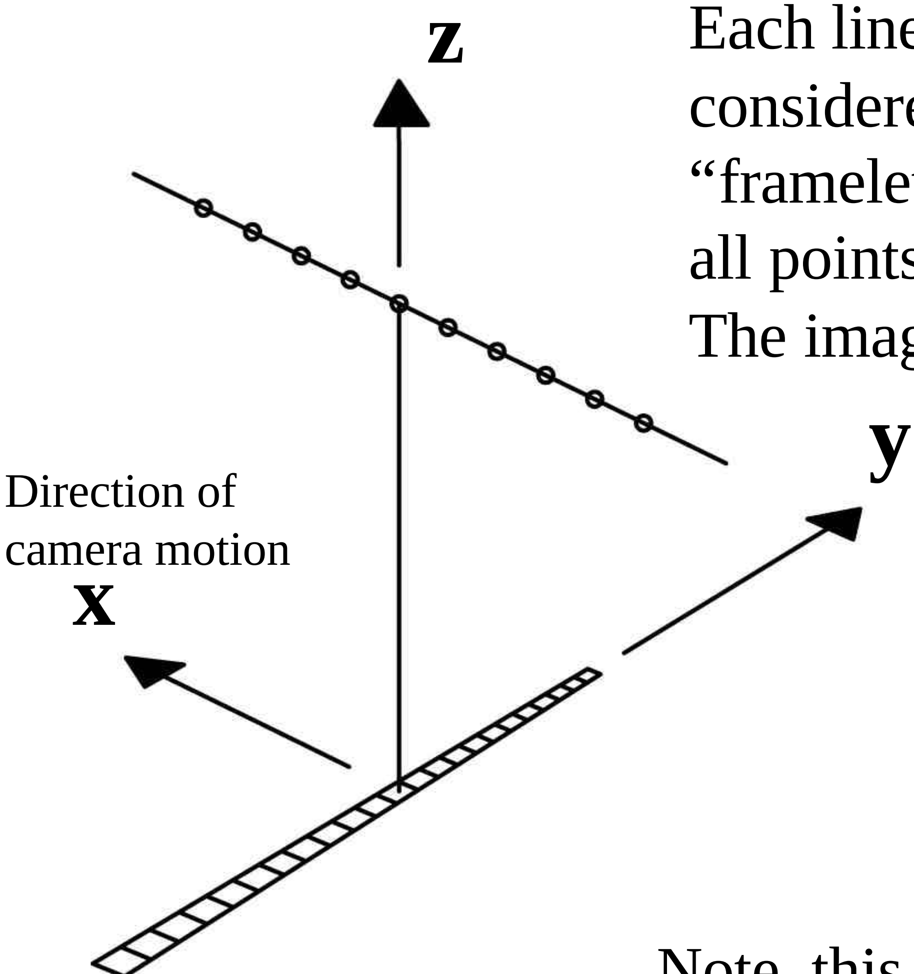
We must also account for a tilt or inclination of the camera. In the case of SPOT this is a cross track tilt (+/- 27 degrees) about the x (motion) axis, implemented by a stationary (but moveable) mirror:

$$\mathbf{M}_{\mathbf{t}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

In the case of an *agile* spacecraft such as IKONOS or Quickbird, this pointing can be any arbitrary cross-track, in-track, or spin attitude, and thus requires 3 rotations:

$$\mathbf{M}_{\mathbf{t}} = \mathbf{M}_{\mathbf{z}}(g)\mathbf{M}_{\mathbf{y}}(b)\mathbf{M}_{\mathbf{x}}(a)$$

Note that we are over parameterized with rotations here. You cannot carry all as unknowns. But it may be convenient to separate in this way to make if clear which physical effect the parameter refers to.



Each line of a pushbroom camera is considered as a separate exposure, a "framelet". As such, within each framelet, all points have an x-coordinate of zero. The image space vector becomes:

$$\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix}$$

Note, this "*f*" is a focal length (not true anomaly)

Collecting all of this into the collinearity condition equation:

$$\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix} = l\mathbf{M_t}\mathbf{M_a}\mathbf{M_b}\left[\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \left[\mathbf{M_b^T}\begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}\right]\right]$$

Image space info → $\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix}$

Scale → l

Rotations → $\mathbf{M_t}\mathbf{M_a}\mathbf{M_b}$

Ground point (ECEF) → $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

Instantaneous exposure station for this line (nominal part from the time and a small correction)

Combine terms, eliminate scale

$$\begin{bmatrix} 0 \\ y \\ -f \end{bmatrix} = l\begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$0 = -f\frac{U}{W} \qquad F_x = f\frac{U}{W}$$

$$y = -f\frac{V}{W} \qquad F_y = y + f\frac{V}{W}$$

We can also add some other inner orientation parameters such as lens distortion, principal point offset, etc.

So how many parameters do we have? There are 5 groups,

- **Orbit parameters** $\Omega, \mathrm{i}, w, \mathrm{a}, \mathrm{e}, \mathrm{t_f}$ (6)
- **Position corrections** $dX_0, dX_1, dX_2, dY_0, dY_1, dY_2, dZ_0, dZ_1, dZ_2$ (9)
- **Attitude corrections** $dw_0, dw_1, dw_2, dj_0, dj_1, dj_2, dk_0, dk_1, dk_2$ (9)
- **Pointing** a_t (1)
- **Inner orientation** $\mathrm{x_0}, \mathrm{y_0}, \mathrm{f}, \mathrm{k_1}$ (4)

Total here is 29, some will be held constant (maybe at zero), we may add some. Stochastic treatment is guided by redundancy, geometric strength of figure (parameters known to be highly correlated will probably not both be carried as unknowns), and by uncertainties

For SPOT we get an approximation of the off-nadir attitude from the angle readout of the mirror position. For Quickbird, we have the attitude described by quaternion elements, throughout the scene.

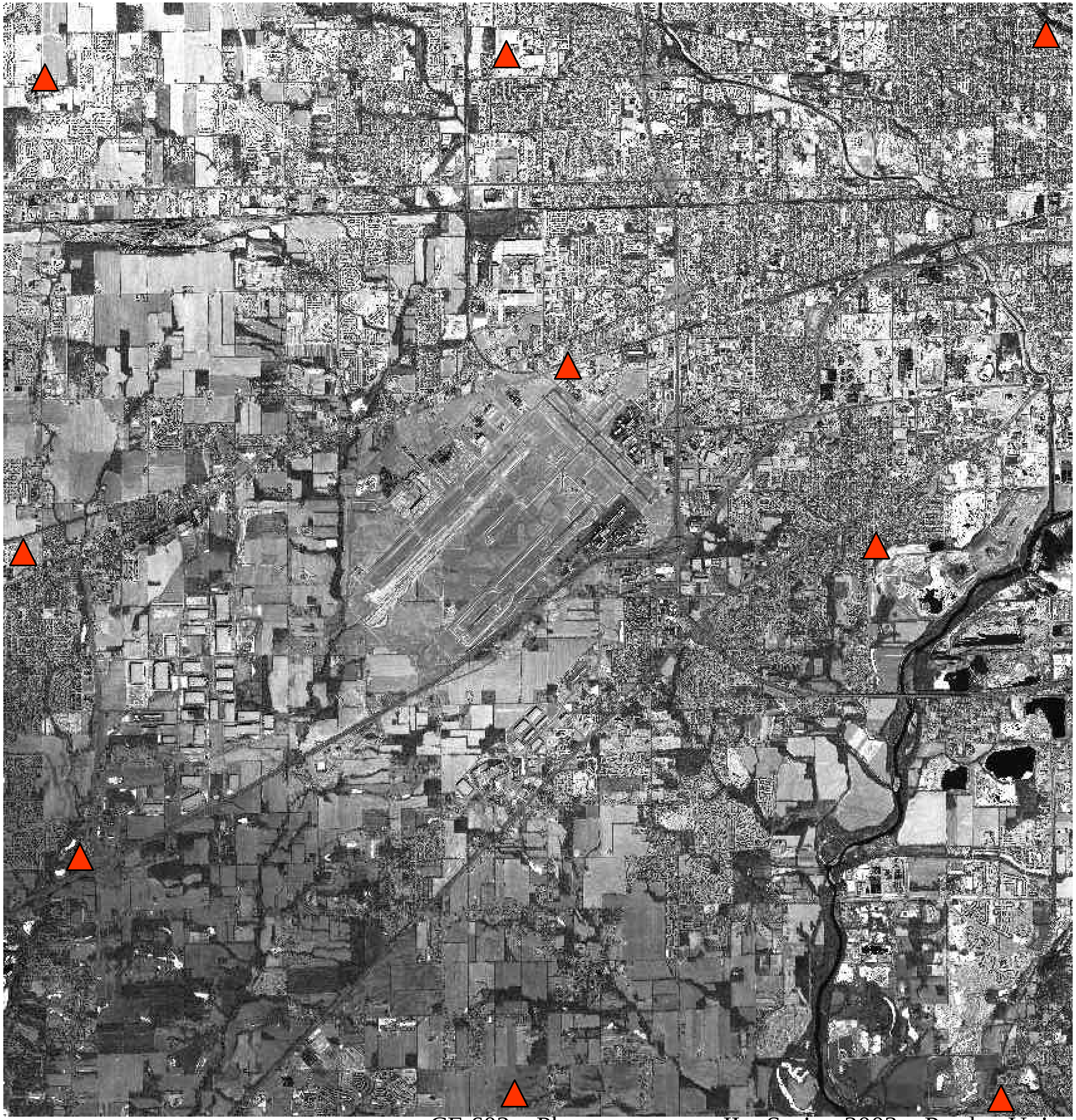
Depending on the source of information about ground control points, we may need to do some prior transformations such as,

$$\begin{bmatrix} X \\ Y \\ H \end{bmatrix} \Rightarrow \begin{bmatrix} j \\ l \\ h \end{bmatrix} \Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Map projection coordinates and orthometric (sea level) height

Geodetic coordinates and ellipsoid height, need info about geoid undulation

Geocentric, ECEF



Quickbird scene (0.6m pixel) over southwestern Indianapolis with 9 “control points” determined with navigation grade GPS (3-5m uncertainty?)

We use these to exercise the resection operation and estimate some of the parameters just described.

Further possibilities: RPC’s, ortho-rectify, drape over terrain for visualizations

Assignment will be put up by Monday for determining some subset(s) of the scene parameters using the given ephemeris data, the given control points, and a MATLAB m-file which implements the just described collinearity model for a generic pushbroom camera. (Read the *product guide* about basic imagery from the DG website, to help in interpretation of support data, and for info about the nominal/design orbit parameters) See spotres3.m and spotceq.m from the textbook software, these will be modified as needed for QB.

(No class Tuesday).

Optional task: determine the RPC coefficients for this scene and compare to those provided by DG. Are the errors in the DG provided support data consistent with the published accuracy figures for “level 1” or “basic” imagery?

Conversion from Position & Velocity to Kepler Elements (Ref: See A. Leick, GPS Satellite Surveying)

$$m = 3.986005E+05$$

Given $\mathbf{X}$ and $\mathbf{V}$, at a given time

$$r = |\mathbf{X}|$$

$$v = |\mathbf{V}|$$

$$\mathbf{H} = \mathbf{X} \times \mathbf{V}$$

$$\mathbf{H} = \begin{bmatrix} h_x \\ h_y \\ h \end{bmatrix}$$

$$h = |\mathbf{H}|$$

$$\boldsymbol{\Omega} = \tan^{-1}\left(\frac{h_x}{-h_y}\right)$$

$$i = \tan^{-1}\left(\frac{\sqrt{h_x^2 + h_y^2}}{h_z}\right)$$

$$R_1(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

$$R_3(\boldsymbol{\Omega}) = \begin{bmatrix} \cos\boldsymbol{\Omega} & \sin\boldsymbol{\Omega} & 0 \\ -\sin\boldsymbol{\Omega} & \cos\boldsymbol{\Omega} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P} = R_1(i)R_3(\boldsymbol{\Omega})\mathbf{X} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$w + f = \tan^{-1}\left(\frac{p_2}{p_1}\right)$$

$$a = \frac{r}{2 - \left(rv^2 / m \right)}$$

$$e = \sqrt{1 - h^2 / ma}$$

$$rv_r = \mathbf{X} \cdot \mathbf{V}$$

$$\sin E = \frac{rv_r}{e\sqrt{ma}}$$

$$\cos E = \frac{(a - r)}{ae}$$

$$f = \tan^{-1}\left(\frac{\sqrt{1 - e^2}\sin E}{\cos E - e} \right)$$

$$E = \tan^{-1}\left(\frac{\sin E}{\cos E} \right)$$

$$M = E - e\sin E$$

$$w = (w + f) - f$$

so now we have,

Ω, i, w, f, a, e

Based on the time associated with each ephemeris point, we can estimate/interpolate the values at the frame center.