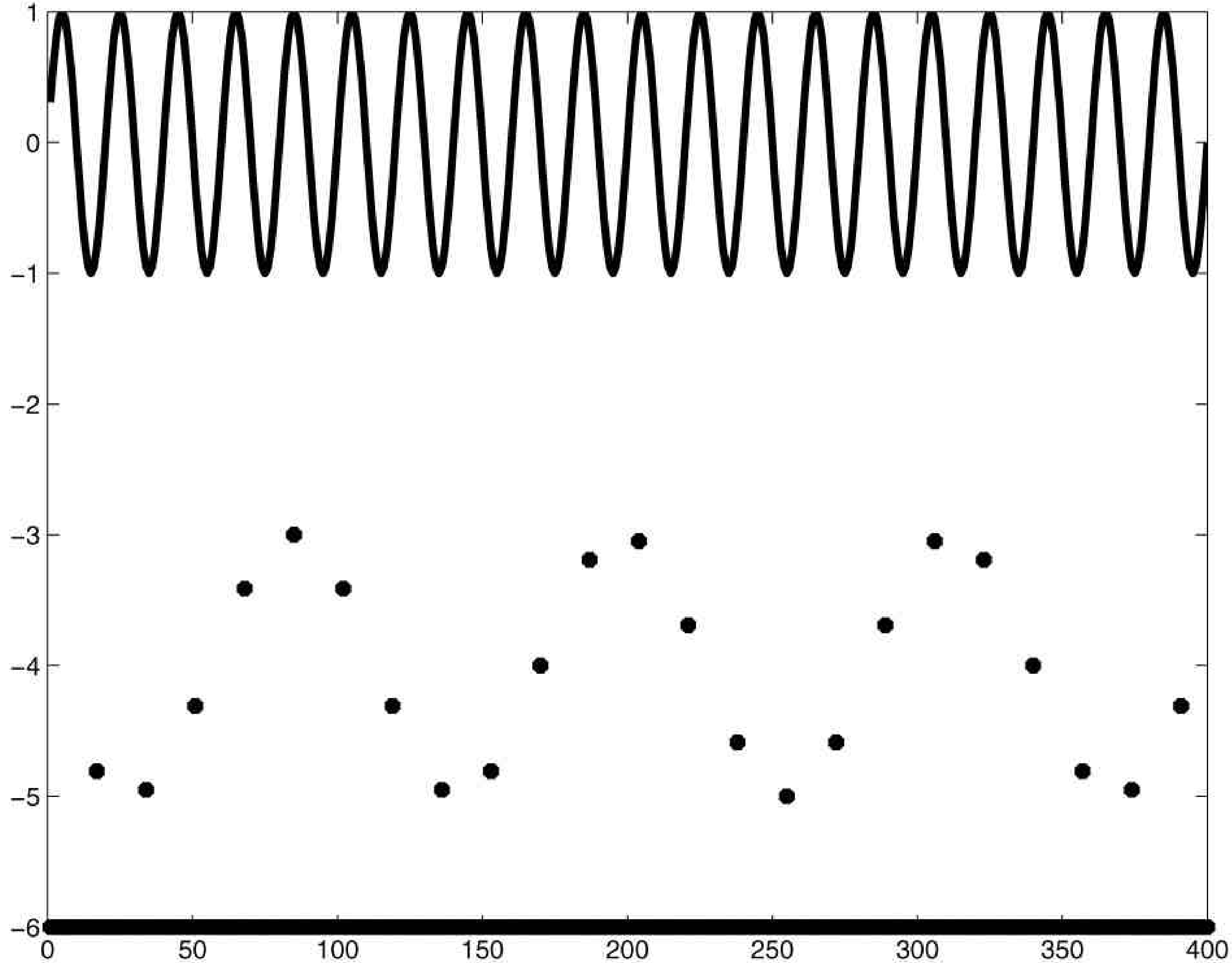
# Sampling and Aliasing

Sampling theorem: If you have a band-limited signal, it can be reconstructed perfectly from samples, provided that you have *at least* two samples per period of the highest frequency present

If you sample at a lower frequency, then when you reconstruct the original signal by interpolation, you will introduce *artifacts* referred to as *aliasing.*

Common example of aliasing: old cowboy movies with stagecoach moving slowly. At slow velocity the frame rate (24fps) is sufficient to capture the spoke motion so it appears correctly. As the stagecoach speed increases and the rotation rate of the wheel increases, at a certain point the movie camera frame rate is not sufficient to capture the wheel motion, and it *appears* to be moving slowly backward, for example. Such artificial pseudo features always become visible in data that is undersampled.

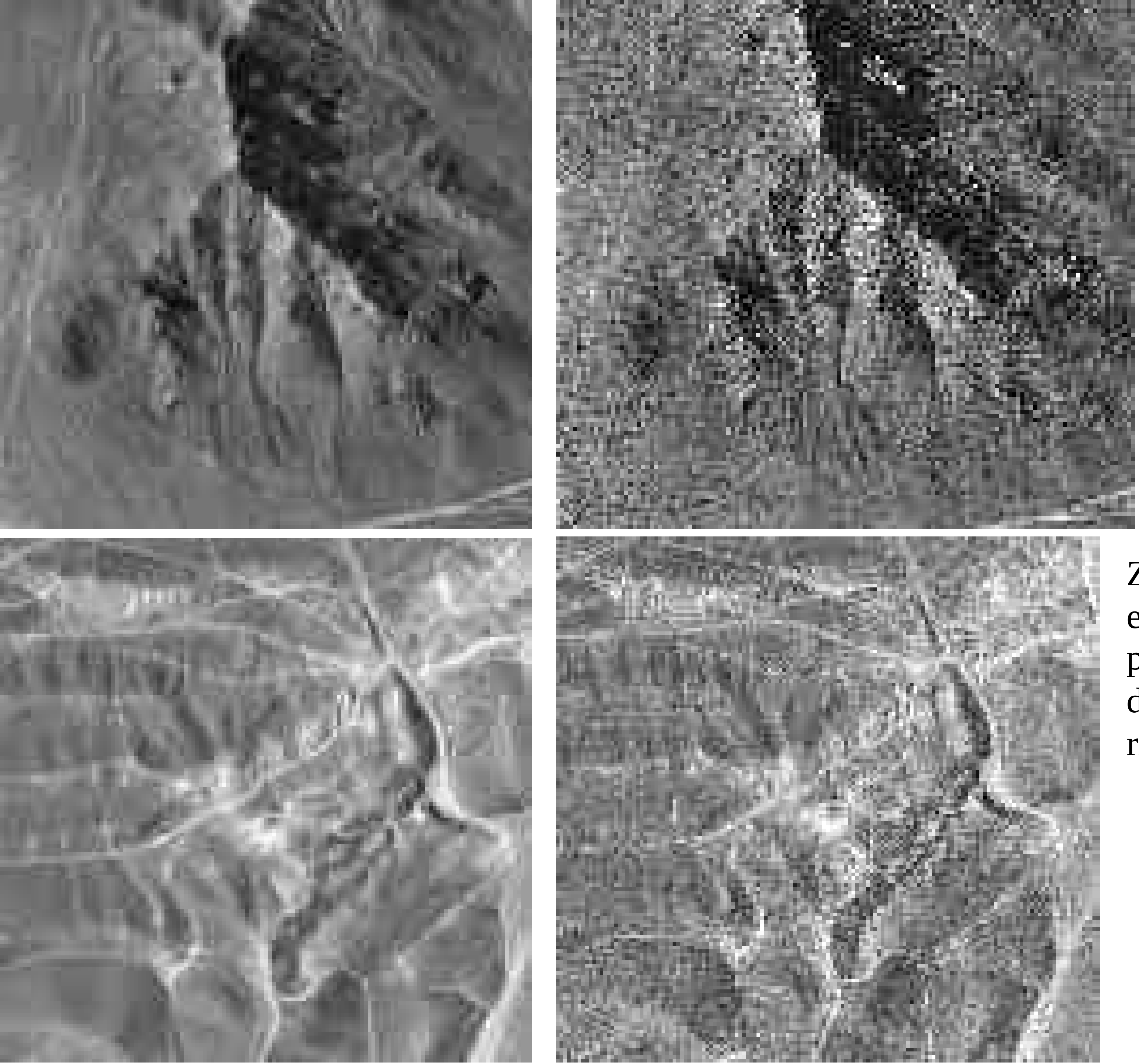
Sine wave sampled at between 1x and 2x per period, yielding the false low frequency data shown below – example of aliasing



# Downsampled Imagery


Properly downsampled


Undersampled



Zoom in on same example, left properly downsampled, right undersampled

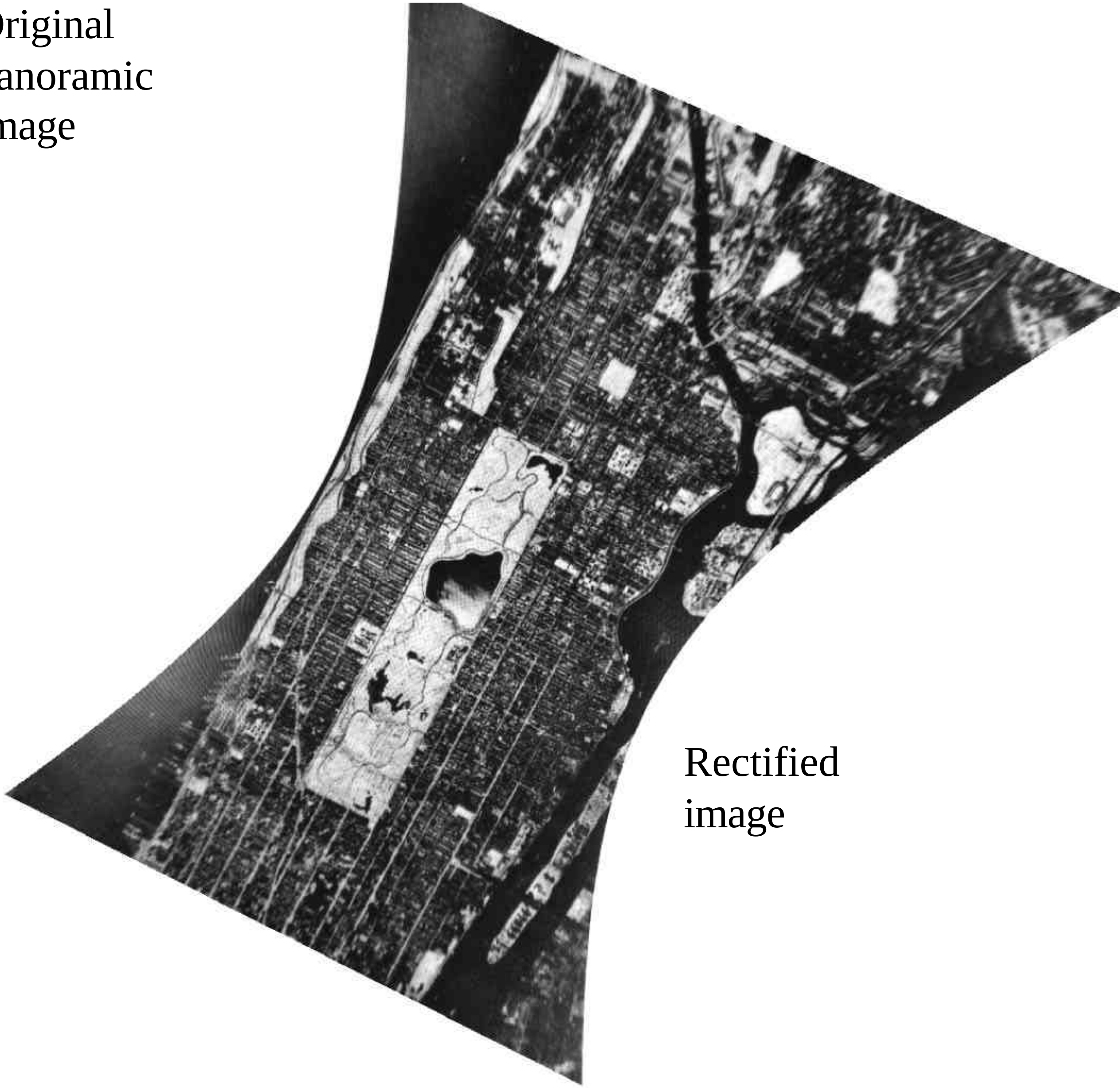
# Solutions

Key insight: Before downsampling, low-pass filter the image so that the (lowered) frequency content matches the sampling interval.

Digital sensor manufacturers often have to consider this: Analog image formed by the optics in the focal plane may have (depending on the quality of the optics) frequencies down to period of 3-5 micrometers. Yet the sampling by a CCD detector may be at 10-20 micrometers, yielding an undersampled image. Usual fix: de-focus the optics to degrade the image sharpness until it just matches the sampling rate of the CCD detetector.

Such solutions work when the sampling rate is uniform – often in rectification we must sample at a non-uniform rate. Then, ideally, your sampling strategy must be dependent on where you are in the image – it is messy.

Original panoramic image



Rectified image

Example of rectification where sampling rate is different in different parts of the image