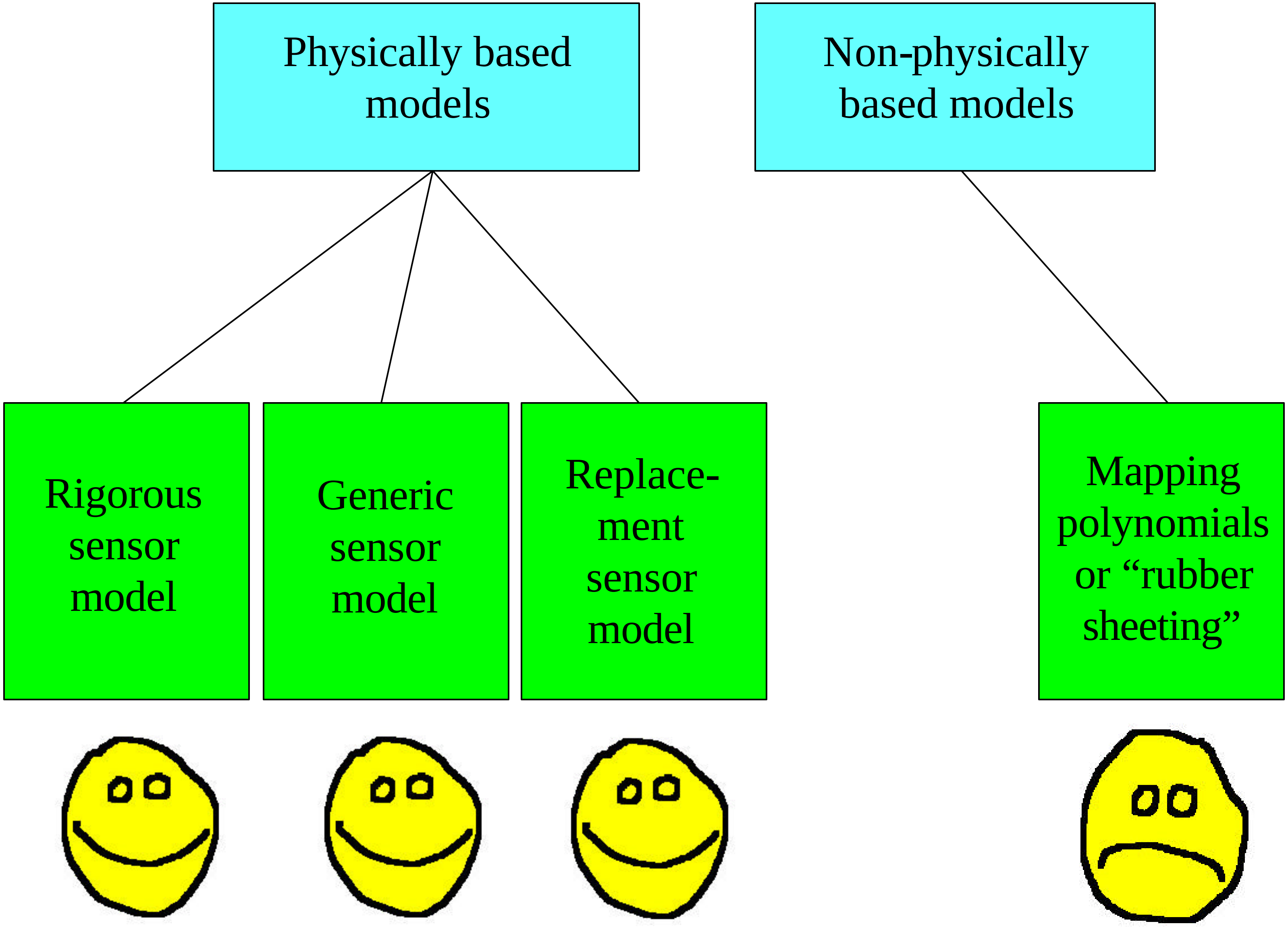
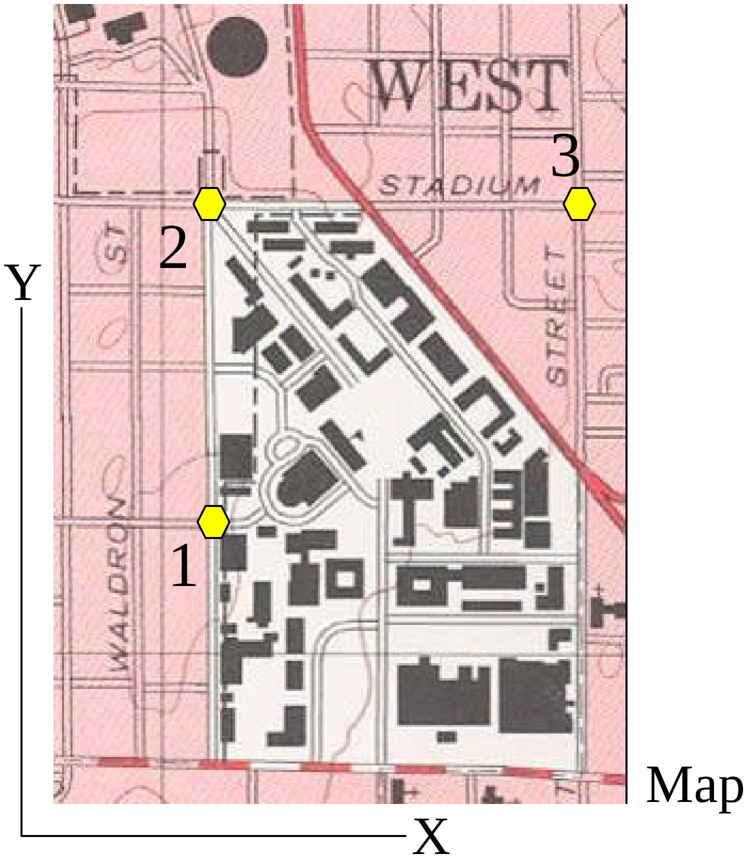
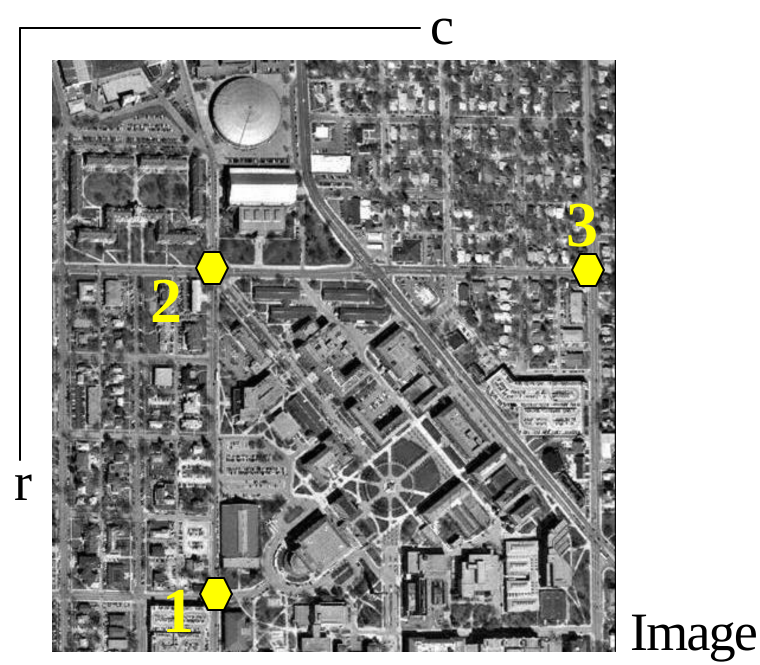
Relating Image and Ground Coordinates



Mapping Polynomials or Rubber Sheeting





For each point we create two equations. We need at least as many equations as unkowns. If more, then we use least squares. It is like a regression problem: linear, easy. But we are confounding the effects of sensor, platform motion, and terrain relief. What should be the order of the polynomial ?

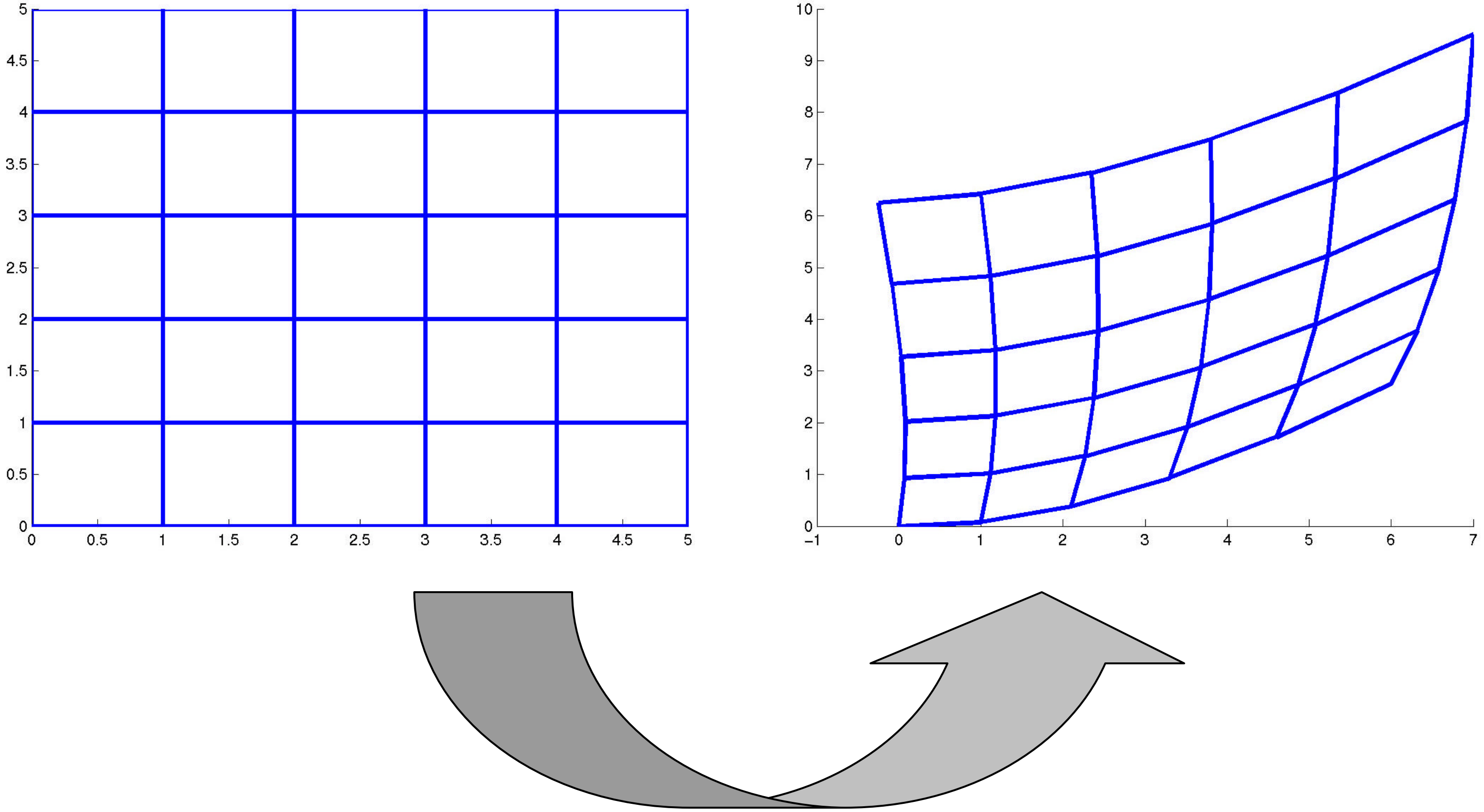
$$r = a_0 + a_1 X + a_2 Y + a_3 XY + a_4 X^2 + a_5 Y^2$$

$$c = b_0 + b_1 X + b_2 Y + b_3 XY + b_4 X^2 + b_5 Y^2$$

Graphical View of Rubber Sheet Transformation (2nd order, 12-parameter)

Reference grid

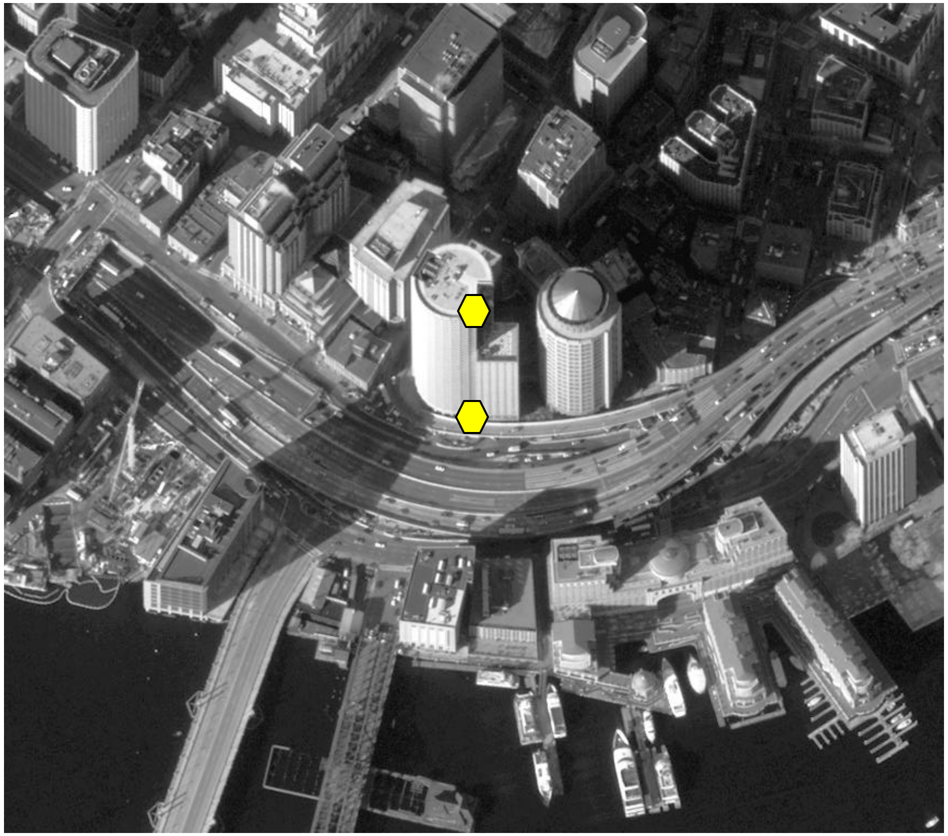
Transformed grid



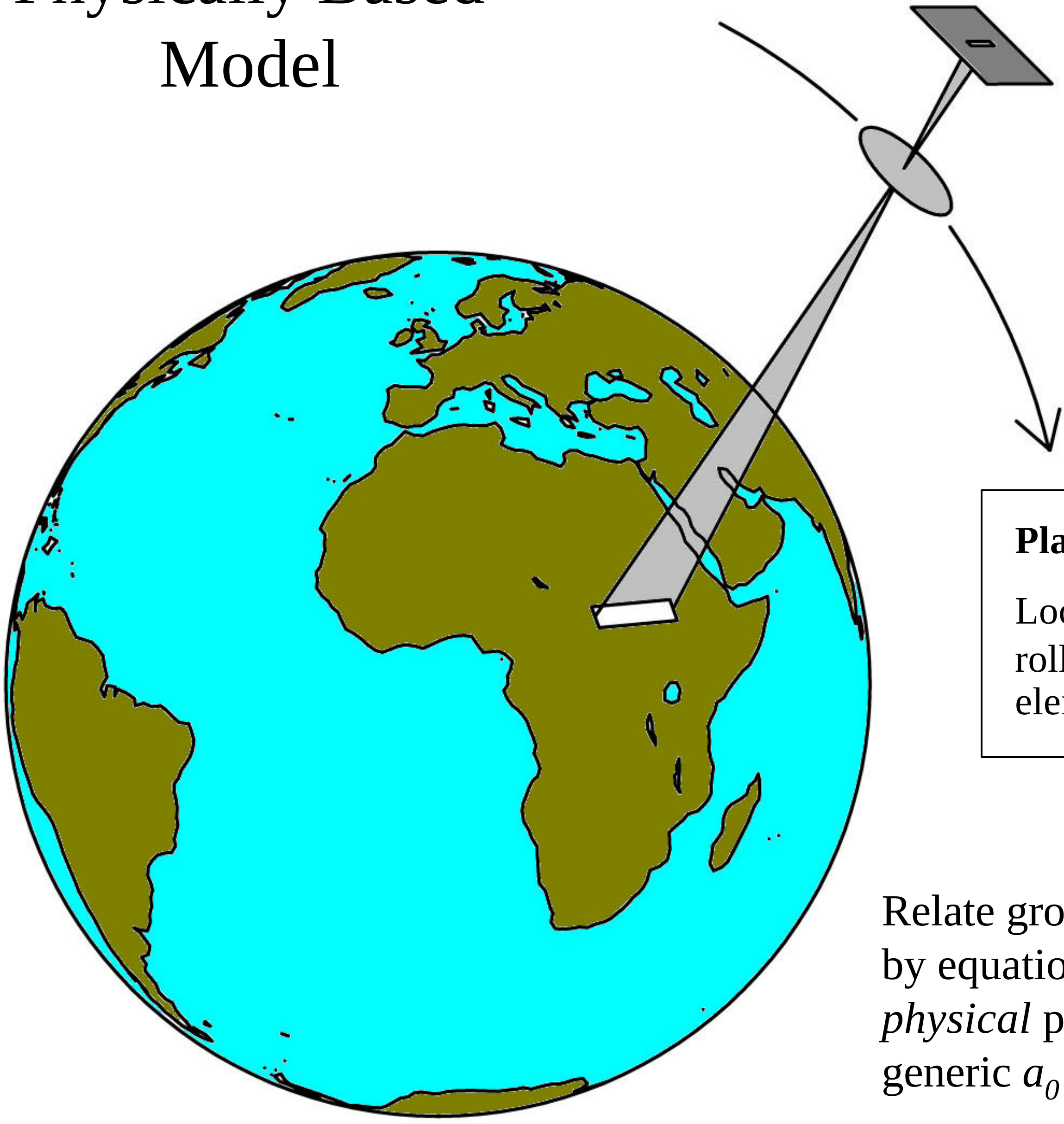
Mapping Polynomials or Rubber Sheeting

If the terrain is flat, the sensor has narrow field of view, the sensor is nadir looking, and the ground sample distance is large, then *you can get reasonable results using the approach of mapping polynomials.*

The accompanying Quickbird image (0.61m pixel) shows the pitfalls of mapping polynomials when the above conditions do not apply. The two marked points have the same XY and they would get mapped into the same (row, col), but clearly that is wrong. You could expand the polynomial by adding some Z-terms. But that would not work. *Modeling the actual physical imaging process is the only way.*



Physically Based Model



Sensor parameters:

Focal length, principal point location, lens distortion, line rate, detector (pixel) size

Platform parameters:

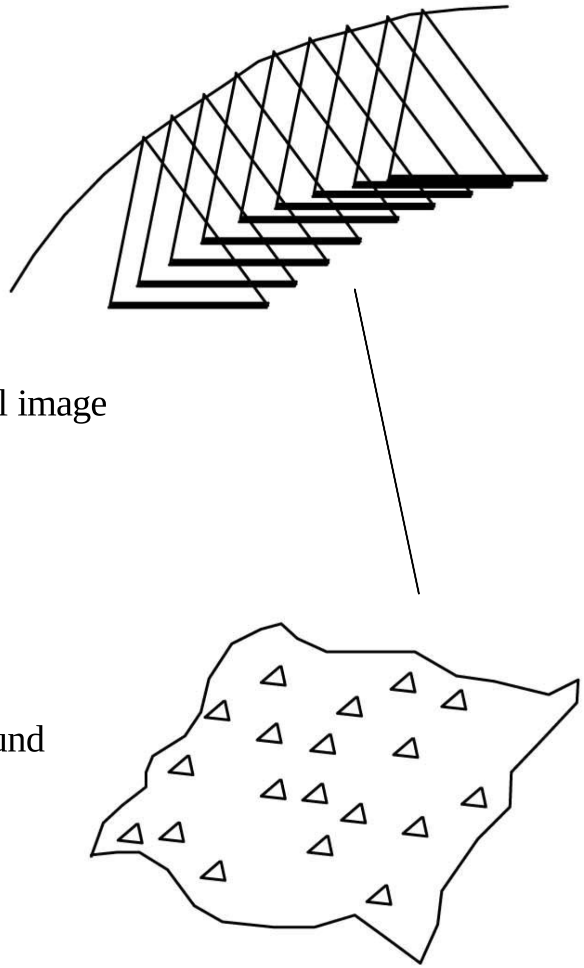
Location X,Y,Z, time, attitude roll, pitch, yaw, kepler orbit elements (*a*,*e*,*i*,**W,w,n**)

Relate ground point and image point by equations with the above *actual physical* parameters, rather than the generic a_0 , a_1 , a_2 , … parameters.

Rigorous Sensor Model Parameter Estimation &

RPC Parameter Estimation

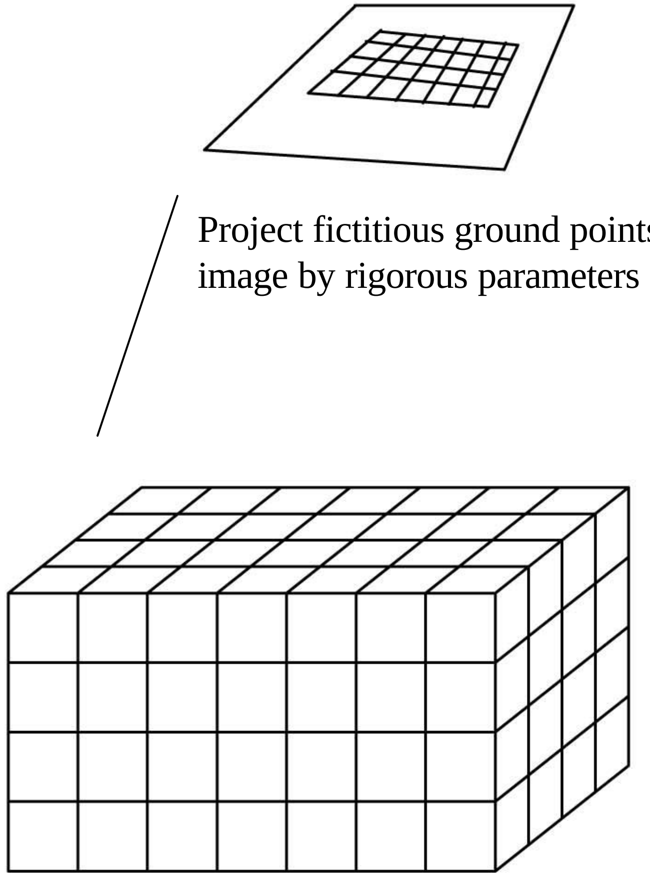
Estimate actual sensor parameters



Actual image points

Actual ground points

Estimate RPC parameters using the *many* fictitious ground and image points



Project fictitious ground points into image by rigorous parameters

Fictitious ground points within volume

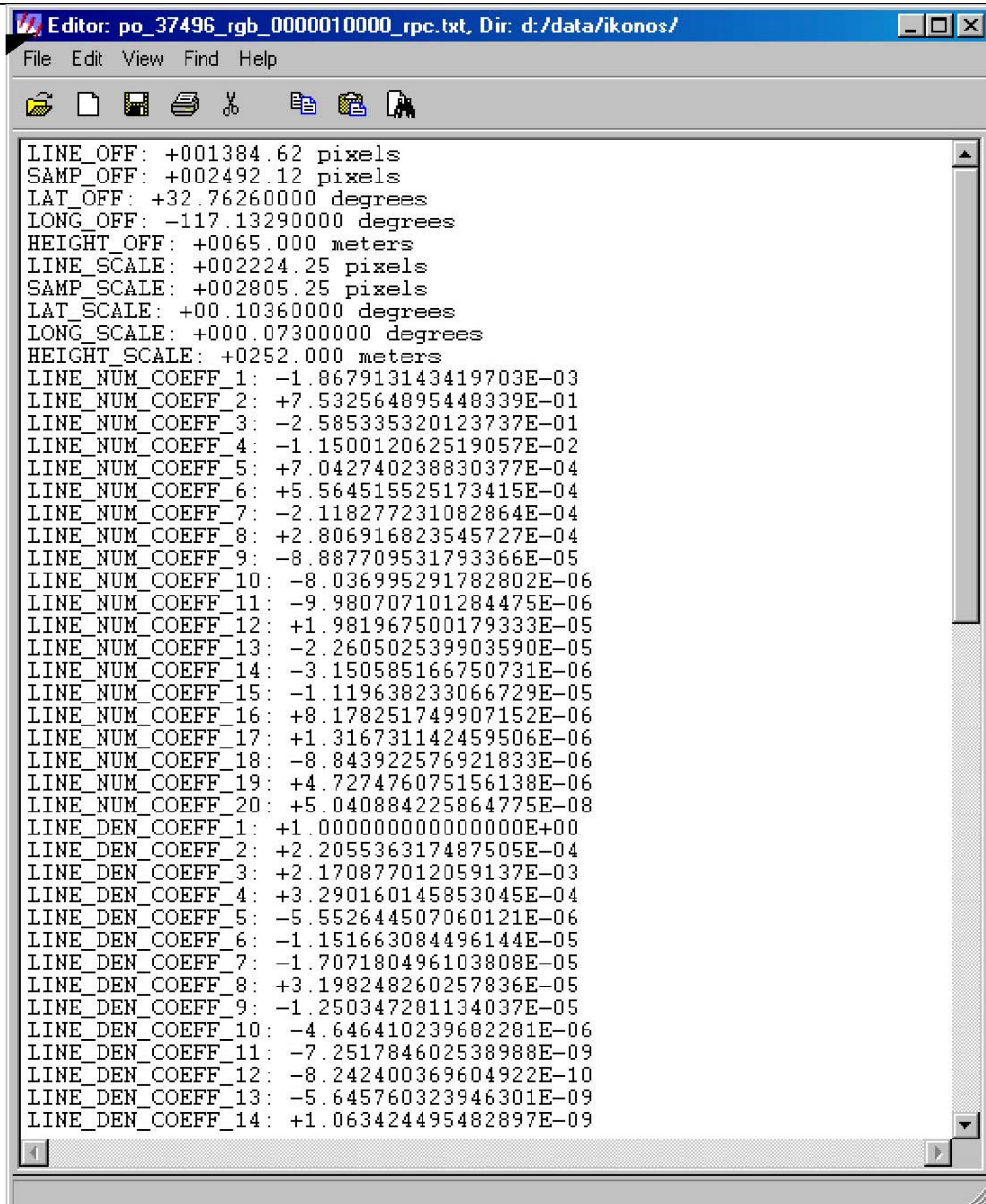
RPC Model

$$r = \frac{p1(X,Y,Z)}{p2(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} a_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} b_{ijk} X^i Y^j Z^k}$$

$$c = \frac{p3(X,Y,Z)}{p4(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} c_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} d_{ijk} X^i Y^j Z^k}$$

For the third order model, only terms with i+j+k <= 3 are allowed. Those terms are shown below.

$$1, x, y, z, x^2, y^2, z^2, xy, xz, yz, x^2y, xy^2, x^2z, xz^2, y^2z, yz^2, x^3, y^3, z^3, xyz$$

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LINE_OFF: +001384.62 pixels
SAMP_OFF: +002492.12 pixels
LAT_OFF: +32.76260000 degrees
LONG_OFF: -117.13290000 degrees
HEIGHT_OFF: +0065.000 meters
LINE_SCALE: +002224.25 pixels
SAMP_SCALE: +002805.25 pixels
LAT_SCALE: +00.10360000 degrees
LONG_SCALE: +000.07300000 degrees
HEIGHT_SCALE: +0252.000 meters
LINE_NUM_COEFF_1: -1.867913143419703E-03
LINE_NUM_COEFF_2: +7.532564895448339E-01
LINE_NUM_COEFF_3: -2.585335320123737E-01
LINE_NUM_COEFF_4: -1.150012062519057E-02
LINE_NUM_COEFF_5: +7.042740238830377E-04
LINE_NUM_COEFF_6: +5.564515525173415E-04
LINE_NUM_COEFF_7: -2.118277231082864E-04
LINE_NUM_COEFF_8: +2.806916823545727E-04
LINE_NUM_COEFF_9: -8.887709531793366E-05
LINE_NUM_COEFF_10: -8.036995291782802E-06
LINE_NUM_COEFF_11: -9.980707101284475E-06
LINE_NUM_COEFF_12: +1.981967500179333E-05
LINE_NUM_COEFF_13: -2.260502539903590E-05
LINE_NUM_COEFF_14: -3.150585166750731E-06
LINE_NUM_COEFF_15: -1.119638233066729E-05
LINE_NUM_COEFF_16: +8.178251749907152E-06
LINE_NUM_COEFF_17: +1.316731142459506E-06
LINE_NUM_COEFF_18: -8.843922576921833E-06
LINE_NUM_COEFF_19: +4.727476075156138E-06
LINE_NUM_COEFF_20: +5.040884225864775E-08
LINE_DEN_COEFF_1: +1.000000000000000E+00
LINE_DEN_COEFF_2: +2.205536317487505E-04
LINE_DEN_COEFF_3: +2.170877012059137E-03
LINE_DEN_COEFF_4: +3.290160145853045E-04
LINE_DEN_COEFF_5: -5.552644507060121E-06
LINE_DEN_COEFF_6: -1.151663084496144E-05
LINE_DEN_COEFF_7: -1.707180496103808E-05
LINE_DEN_COEFF_8: +3.198248260257836E-05
LINE_DEN_COEFF_9: -1.250347281134037E-05
LINE_DEN_COEFF_10: -4.646410239682281E-06
LINE_DEN_COEFF_11: -7.251784602538988E-09
LINE_DEN_COEFF_12: -8.242400369604922E-10
LINE_DEN_COEFF_13: -5.645760323946301E-09
LINE_DEN_COEFF_14: +1.063424495482897E-09
```

Erdas Imagine / Orthobase support for IKONOS RPC data – note the line_numerator coefficients go up to #20, this implies a 3rd order polynomial

$$r = \frac{a_0 + a_1 x + a_2 y}{1 + c_1 x + c_2 y}$$

$$c = \frac{b_0 + b_1 x + b_2 y}{1 + c_1 x + c_2 y}$$

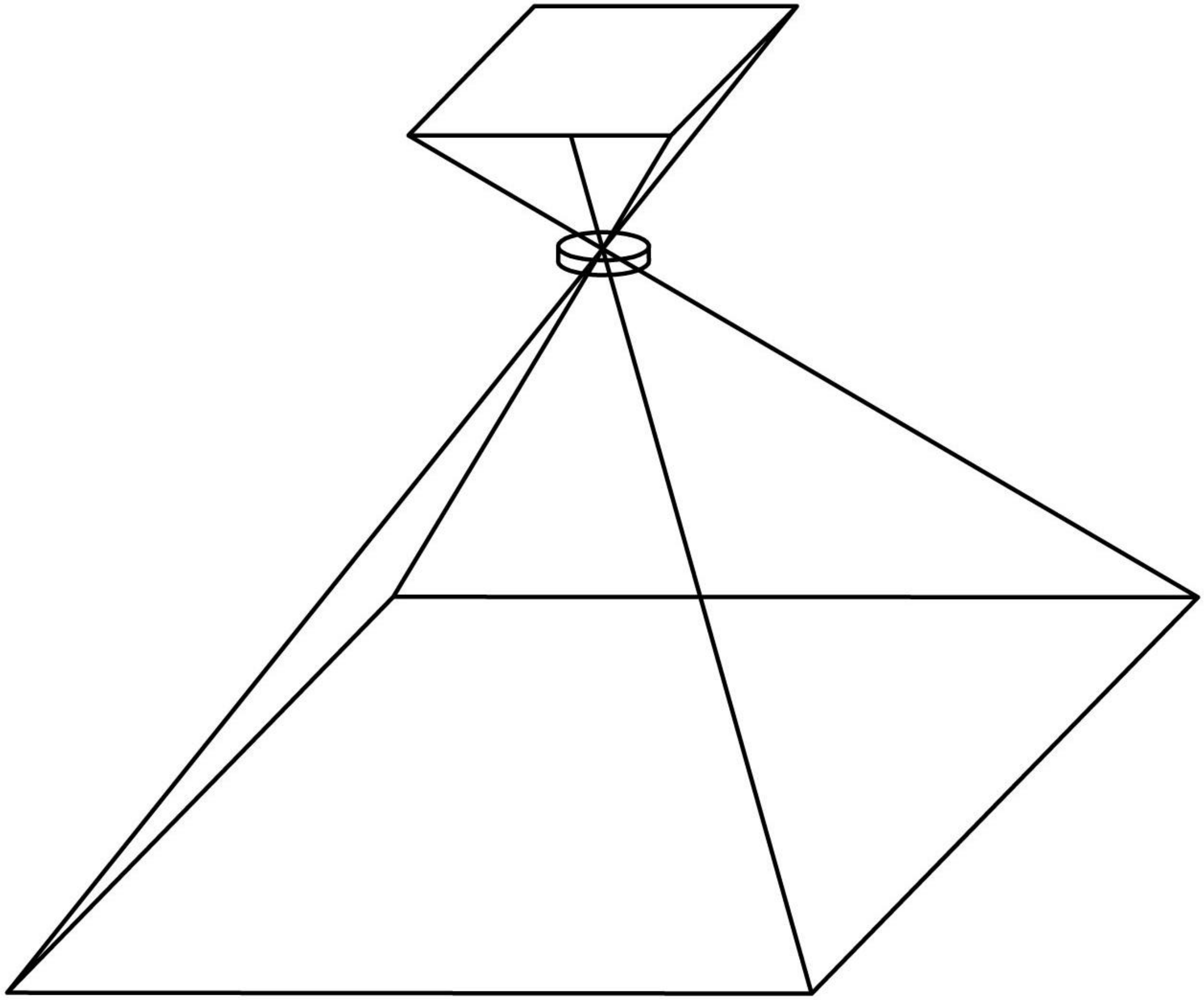
$$r + rc_1 x + rc_2 y = a_0 + a_1 x + a_2 y$$
$$c + cc_1 x + cc_2 y = b_0 + b_1 x + b_2 y$$

$$r = a_0 + a_1 x + a_2 y - rc_1 x - rc_2 y$$
$$c = b_0 + b_1 x + b_2 y - cc_1 x - cc_2 y$$

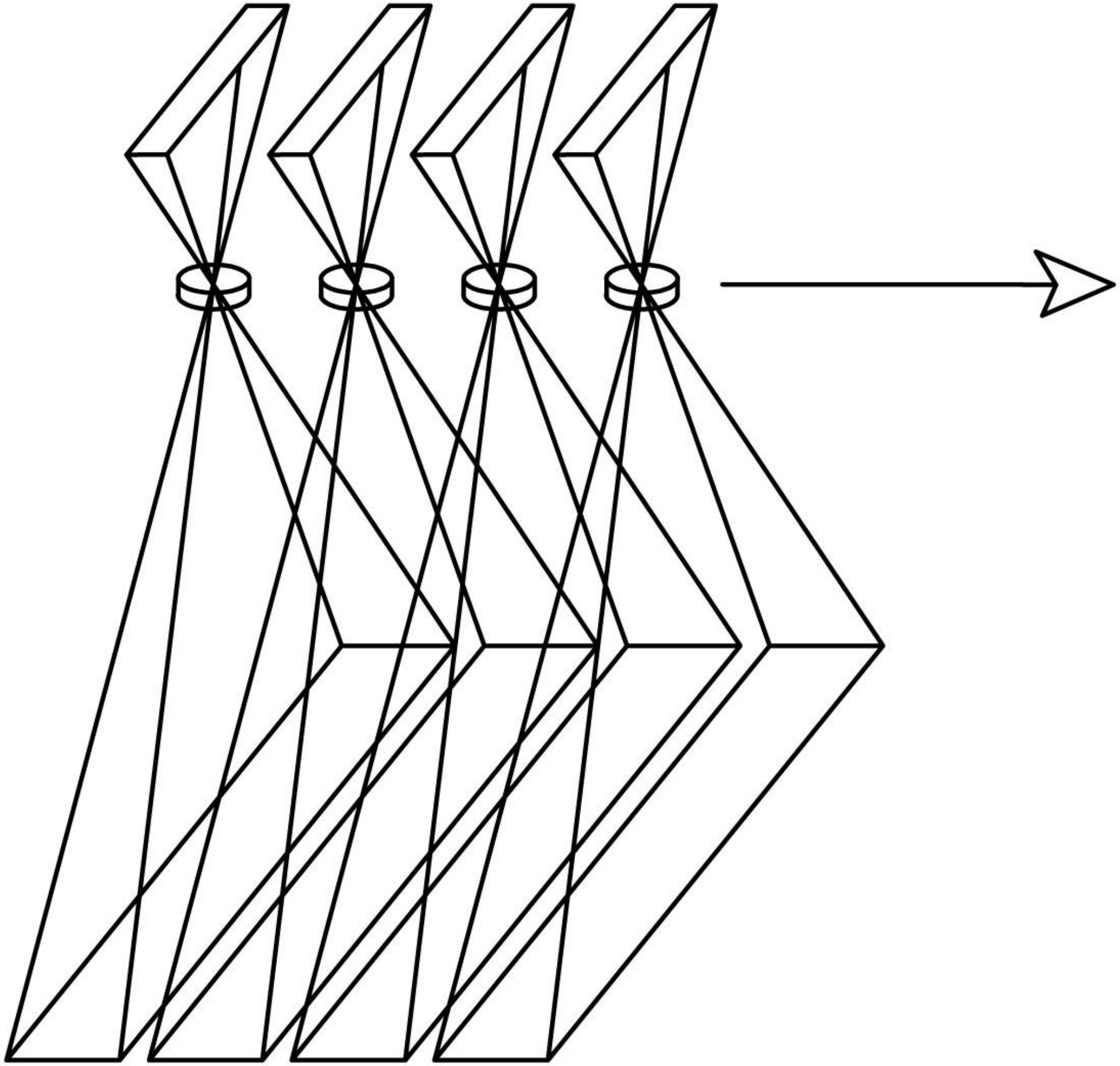
$$\begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 & -rx & -ry \\ 0 & 0 & 0 & 1 & x & y & -cx & -cy \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

Show handling of low order rational polynomials as pseudo-linear problem. This is a good way to get approximations for the parameters, then final estimates can be obtained by rigorous non-linear estimation. Note that we will scale both the object and image coordinates into the range: -1 to +1.

Frame (Pinhole Camera) Sensor Geometry



Pushbroom Sensor Geometry



Panoramic Sensor Geometry