(LS)

Least Squares for The Satellite Photogrammetry Resection

condition equations $F_\phi(l,s,h,\phi,\lambda;dp) = 0$

$$F_\lambda(l,s,h,\phi,\lambda;dp) = 0$$

The equations just say if we project the point l,s down to the ground, the misclosure in ϕ,λ should be zero. After adjustment it will not be zero, but it will be small. $dp = \begin{bmatrix} dq_i \\ dq_j \\ dq_k \\ dq_s \end{bmatrix}$ is the vector of unknown parameters, corrections to the quaternion elements interpolated at each control point. dp initially is the zero vector.

For nonlinear LS we need the partial derivatives of the condition equations with respect to the observations, and with respect to the unknown parameters (dp). We evaluate these partials by numerical approximation.

$$\frac{\partial F_\phi}{\partial l} \approx \frac{\partial F_\phi(l+\Delta l, s, h, \phi, \lambda, dp) - \partial F_\phi(l,s,h,\phi,\lambda,dp)}{\Delta l}, \text{ and}$$

$$\frac{\partial F_\lambda}{\partial l} \approx \frac{\partial F_\lambda(l+\Delta l, s, h, \phi, \lambda, dp) - \partial F_\lambda(l,s,h,\phi,\lambda,dp)}{\Delta l}, \text{ etc.}$$

Same idea for $\frac{\partial F_\phi}{\partial dq_i}$, $\frac{\partial F_\lambda}{\partial dq_i}$, etc.

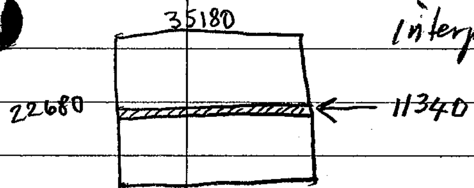
structure of condition equations:

$$\underbrace{\begin{bmatrix} \frac{\partial F_{\phi_1}}{\partial l} & \frac{\partial F_{\phi_1}}{\partial s} & & & \\ \frac{\partial F_{\lambda_1}}{\partial l} & \frac{\partial F_{\lambda_1}}{\partial s} & & & \\ & & \frac{\partial F_{\phi_2}}{\partial l} & \frac{\partial F_{\phi_2}}{\partial l} & \\ & & \frac{\partial F_{\lambda 2}}{\partial l} & \frac{\partial F_{\lambda 2}}{\partial s} & \\ & & & & \ddots \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_{l_1} \\ V_{s_1} \\ V_{l_2} \\ V_{s_2} \\ \vdots \end{bmatrix}}_{V} + \underbrace{\begin{bmatrix} \frac{\partial F_{\phi_1}}{\partial dq_i} & \frac{\partial F_{\phi_1}}{\partial dq_j} & \frac{\partial F_{\phi_1}}{\partial dq_k} & \frac{\partial F_{\phi_1}}{\partial dq_s} \\ \frac{\partial F_{\lambda_1}}{\partial dq_i} & \frac{\partial F_\lambda}{\partial dq_j} & \frac{\partial F_\lambda}{\partial dq_k} & \frac{\partial F_\lambda}{\partial dq_s} \\ & & \vdots & \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \Delta dq_i \\ \Delta dq_j \\ \Delta dq_k \\ \Delta dq_s \end{bmatrix}}_{\Delta} = \underbrace{\begin{bmatrix} -F_{\phi_1} \\ -F_{\lambda_1} \\ -F_{\phi_2} \\ -F_{\lambda_2} \\ \vdots \end{bmatrix}}_{f}$$

$$A \quad V + B \quad \Delta = f$$

we will evaluate partials and right side vectors at the original observations. Use Matlab function fi2g_pl_ϕ_part(l,s,h,ϕ,λ,dp) to get A, B, f.

to impose the constraint that the adjusted quaternions should have length = 1, we choose a representative attitude quaternion in middle of image.



interpolate q_m at line 11340 (middle of image)

then write the constraint equation:

$$(q_{m_i} + dq_i)^2 + (q_{m_j} + dq_j)^2 + (q_{m_k} + dq_k)^2 + (q_{m_s} + dq_s)^2 = 1$$

OR

$$F_c = (q_{m_i} + dq_i)^2 + (q_{m_j} + dq_j)^2 + (q_{m_k} + dq_k)^2 + (q_{m_s} + dq_s)^2 - 1 = 0$$

Linearize this into the form $C\Delta = g$

$$C = \left[\frac{\partial F_c}{\partial dq_i} \quad \frac{\partial F_c}{\partial dq_j} \quad \frac{\partial F_c}{\partial dq_k} \quad \frac{\partial F_c}{\partial dq_s}\right], \quad \Delta = \begin{bmatrix} \delta dq_i \\ \delta dq_j \\ \delta dq_k \\ \delta dq_s \end{bmatrix}, \quad g = -F_c$$

handle by direct method where N is full rank:

$$k_c = (CN^{-1}C^T)^{-1}(g - CN^{-1}t)$$
$$\Delta = N^{-1}t + N^{-1}C^T k_c$$

where $Q_e = AQA^T$, $Q = W^{-1}$, $W_e = Q_e^{-1}$

$N = B^T W_e B$, $t = B^T W_e f$

for each iteration compute $A, B, f, C, g, k_c, \Delta$.

update $[dq_i \; dq_j \; dq_k \; dq_s]^T$, until Δ is not significant

when converged,

$$k = W_e (f - B\Delta)$$
$$v = QA^T k$$

residuals should be ≈ ½ pixel

see my post in notes about updated heights for control points