Nonlinear KF

observation $\left\{ F_{obs}(x_i) = 0 \right.$; we will linearize this about some value x^0, see options

state transition $\left\{ \begin{array}{l} x_i = F_{Trans}(x_{i-1}) \text{ ; we only need to evaluate this - not to linearize it} \\ \text{evaluate @ } x^0 \text{ of epoch } i-1 \\ \text{if } x_i = \Phi x_{i-1} \text{ is already linear then just evaluate it without need to consider } x^0 \end{array} \right.$

So for epoch i we have the linearized observation equations

$$F_{obs}(x_i) \approx F^0_{obs}(x^0) + \left[\frac{\partial F_{obs}}{\partial x}\right]_{x=x^0} [\Delta x] = 0$$

and the "unified LS" equation relating the new parameters, x, to the a priori values, x^-,

$$x_i = x_i^- , \quad \text{or} \quad I x_i = x_i^-$$

The weight matrices for these two sets of equations are respectively R^{-1} and $(P^-)^{-1}$

Linearizing the second equation,

$$F_{ULS}(x_i) = I x_i - x_i^- = 0$$

$$F_{ULS}(x_i) \approx F^0_{ULS}(x^0) + \left[\frac{\partial F_{ULS}}{\partial x}\right]_{x=x^0} (\Delta x) = 0$$

combining the linearized observation equations and the linearized unified equations, we get,

$$\left[\frac{\partial F_{obs}}{\partial x}\right]_{x=x_i^0} [\Delta x] = - F^0_{obs}(x^0)$$

$$\left[\frac{\partial F_{ULS}}{\partial x}\right]_{x=x^0} (\Delta x) = - F^0_{ULS}(x^0) = x^- - x^0$$

this is $-f_x$ recall $f_x = x^0 - x$ from OLS p. 341

or,

$$\begin{bmatrix} H_{X^0} \\ I \end{bmatrix} [\Delta X] = \begin{bmatrix} -F_{obs}(X^0) \\ -F_{KLS}(X^0) \end{bmatrix} \quad \text{with} \quad W = \begin{bmatrix} R^{-1} & 0 \\ 0 & (P^-)^{-1} \end{bmatrix}$$

at what value of the $\underbrace{\text{state vector}}_{X^0}$ do we linearize? <u>3 choices</u>

1. X^0 from reference trajectory X_{ref}
   compute ΔX once, $X_{new} = X^0 + \Delta X$
   called "Linearized Kalman Filter", LKF

2. X^0 from X^- (i.e. predicted from prior KF step)
   compute ΔX once $X_{new} = X^0 + \Delta X$
   called "Extended Kalman Filter", EKF

3. X^0 from X^- (as before)
   compute ΔX <u>iteratively</u> until convergence, $X_{new} = X^0 + \Delta X$
   called "Iterated Extended Kalman Filter", IEKF

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at each epoch, after evolving the state vector (full coordinates)
$X_i^- = \Phi_{i-1} X_{i-1}$, then the Kalman update <u>variables</u> take <u>new meaning</u> for <u>non linear</u> case:

$X^- \Rightarrow \Delta X^- = X^-_{full} - X^0_{full}$

H = jacobian of obs eqn's wrt state var evaluated @ $X = X^0$ $\quad \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \frac{\partial F_1}{\partial X_2} & \cdots \\ \frac{\partial F_2}{\partial X_1} & \frac{\partial F_2}{\partial X_2} & \cdots \\ \vdots & & \end{bmatrix}$

$Z = -F$; misclosure of obs. eqn's evaluated @ $X = X^0$

$X \Rightarrow \Delta X$ = correction applied to X^0,

Summary

Linear

1. $$\begin{cases} X_i = \Phi_{i-1} X_{i-1} \\ P_i^- = \Phi_{i-1} P_{i-1} \Phi_{i-1}^T + Q \end{cases}$$

2. $K = P^- H^T (R + H P^- H^T)^{-1}$

3. $X = X^- + K(Z - H X^-)$

4. $P = (I - KH) P^-$

Non Linear

1. $$\begin{cases} X_i = \Phi_{i-1} X_{i-1} \\ P_i^- = \Phi_{i-1} P_{i-1} \Phi_{i-1}^T + Q \end{cases}$$

2. $K = P^- H^T (R + H P^- H^T)^{-1}$

3. $\Delta X = \Delta X^- + K(Z - H \Delta X^-)$

4. $P = (I - KH) P^-$

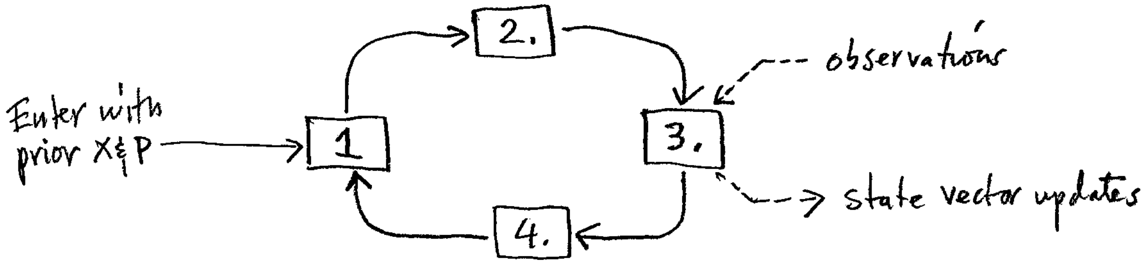
equations linearized at X°

$$X = X^\circ + \Delta X$$

$$H = \left[\frac{\partial F_{obs}}{\partial X}\right]_{X=X^\circ}$$

$$Z = \left[-F_{obs}\right]_{X=X^\circ}$$

$$\Delta X^- = X^- - X^\circ$$



Kalman Filter Loop