$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

1a) $A_{11} B_{11} + A_{12} B_{21} = I$

2a) $A_{11} B_{12} + A_{12} B_{22} = 0$

3a) $A_{21} B_{11} + A_{22} B_{21} = 0$

4a) $A_{21} B_{12} + A_{22} B_{22} = I$

Solve 3a for B_{21}

$$\boxed{B_{21} = -A_{22}^{-1} A_{21} B_{11}}$$

plug into 1a

$$A_{11} B_{11} - A_{12} A_{22}^{-1} A_{21} B_{11} = I$$

$$(A_{11} - A_{12} A_{22}^{-1} A_{21}) B_{11} = I$$

$$\boxed{B_{11} = (A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1}}$$

from 4a

$$\boxed{B_{22} = A_{22}^{-1} - A_{22}^{-1} A_{21} B_{12}}$$

from 2b

$$B_{11} A_{12} + B_{12} A_{22} = 0$$

$$B_{12} A_{22} = -B_{11} A_{12}$$

$$\boxed{B_{12} = -B_{11} A_{12} A_{22}^{-1}}$$

$\boxed{1}$

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

1b) $B_{11} A_{11} + B_{12} A_{21} = I$

2b) $B_{11} A_{12} + B_{12} A_{22} = 0$

3b) $B_{21} A_{11} + B_{22} A_{21} = 0$

4b) $B_{21} A_{12} + B_{22} A_{22} = I$

Solve 2a for B_{12}

$$\boxed{B_{12} = -A_{11}^{-1} A_{12} B_{22}}$$

plug into 4a

$$-A_{21} A_{11}^{-1} A_{12} B_{22} + A_{22} B_{22} = I$$

$$(A_{22} - A_{21} A_{11}^{-1} A_{12}) B_{22} = I$$

$$\boxed{B_{22} = (A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1}}$$

from 1a

$$\boxed{B_{11} = A_{11}^{-1} - A_{11}^{-1} A_{12} B_{21}}$$

from 3b

$$B_{21} A_{11} + B_{22} A_{21} = 0$$

$$B_{21} A_{11} = -B_{22} A_{21}$$

$$\boxed{B_{21} = -B_{22} A_{21} A_{11}^{-1}}$$

$\boxed{2}$

Matrix inverse by partitioning & derivation of Sherman Morrison Woodbury Schur Formula

9 Nov. 2017

$$B_{11} = \overset{\boxed{1}}{(A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1}} = \overset{\boxed{2}}{A_{11}^{-1} - A_{11}^{-1} A_{12} B_{21}} = \overset{\boxed{2}}{A_{11}^{-1} + A_{11}^{-1} A_{12} B_{22} A_{21} A_{11}^{-1}}$$

$$(A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1} = A_{11}^{-1} + A_{11}^{-1} A_{12} (A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1} A_{21} A_{11}^{-1} \quad \boxed{2}$$

$$(Y - U Z V)^{-1} = Y^{-1} + Y^{-1} U (Z^{-1} - V Y^{-1} U)^{-1} V Y^{-1}$$

$(U \leftarrow -U)$ $$\boxed{(Y + U Z V)^{-1} = Y^{-1} - Y^{-1} U (Z^{-1} + V Y^{-1} U)^{-1} V Y^{-1}}$$ ✓