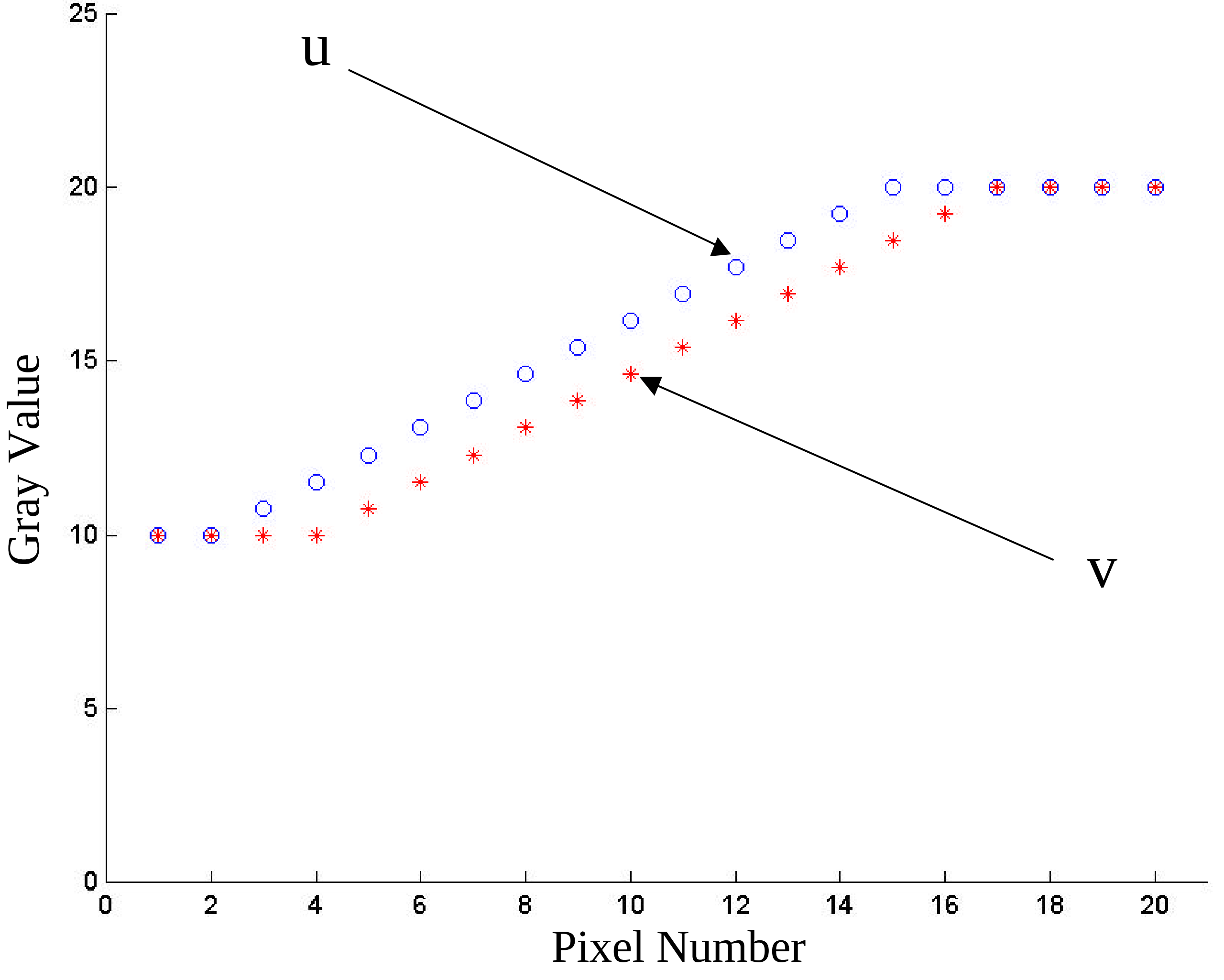
Exam Review

- Exam Friday, 29 march, 3:30
- Closed book
- I will return homeworks on Thursday
- Some short answer definitions, some problems with computation, (bring calculator)
- Review everything on the course web page, plus textbook chapters 1,2,3, 4(collinearity & orbital case), and second half of appdx. F
- Definitions & concepts: field of view, IFOV, GSD, active, passive, frame, pushbroom, scanner, panoramic, panchromatic, multispectral, hyperspectral, optics, filtering, swath, rigorous sensor model, approximate sensor model, orbits for remote sensing, kepler elements, ccd operation, collinearity equations (coords & angles), sequential rotations, linear model vs. nonlinear model & tricks, relief displacement, ellipsoid height vs. orthometric height, solution singularities: shift vs. rotation, simple rectification, orthorectification, interpolation, aliasing

Least Squares Matching - 1D Tutorial



Carry one (horizontal) shift parameter, and estimate this shift by minimizing the corrections to the v-sequence. It is nonlinear so linearize in the usual way and make iterative solution.

Appproach

•We will carry one horizontal shift parameter

•Could carry also a horizontal scale parameter, a vertical shift parameter, or a vertical scale parameter (see following sketches)

•Will write one condition (observation) equation per pixel

•Think of the horizontal axis as pixel number , and the vertical axis as gray level

•The condition equation will be a nonlinear function of the shift parameter

•Linearize by the usual taylor series approach

•This will require solving for a *correction* rather than the actual *parameter value*

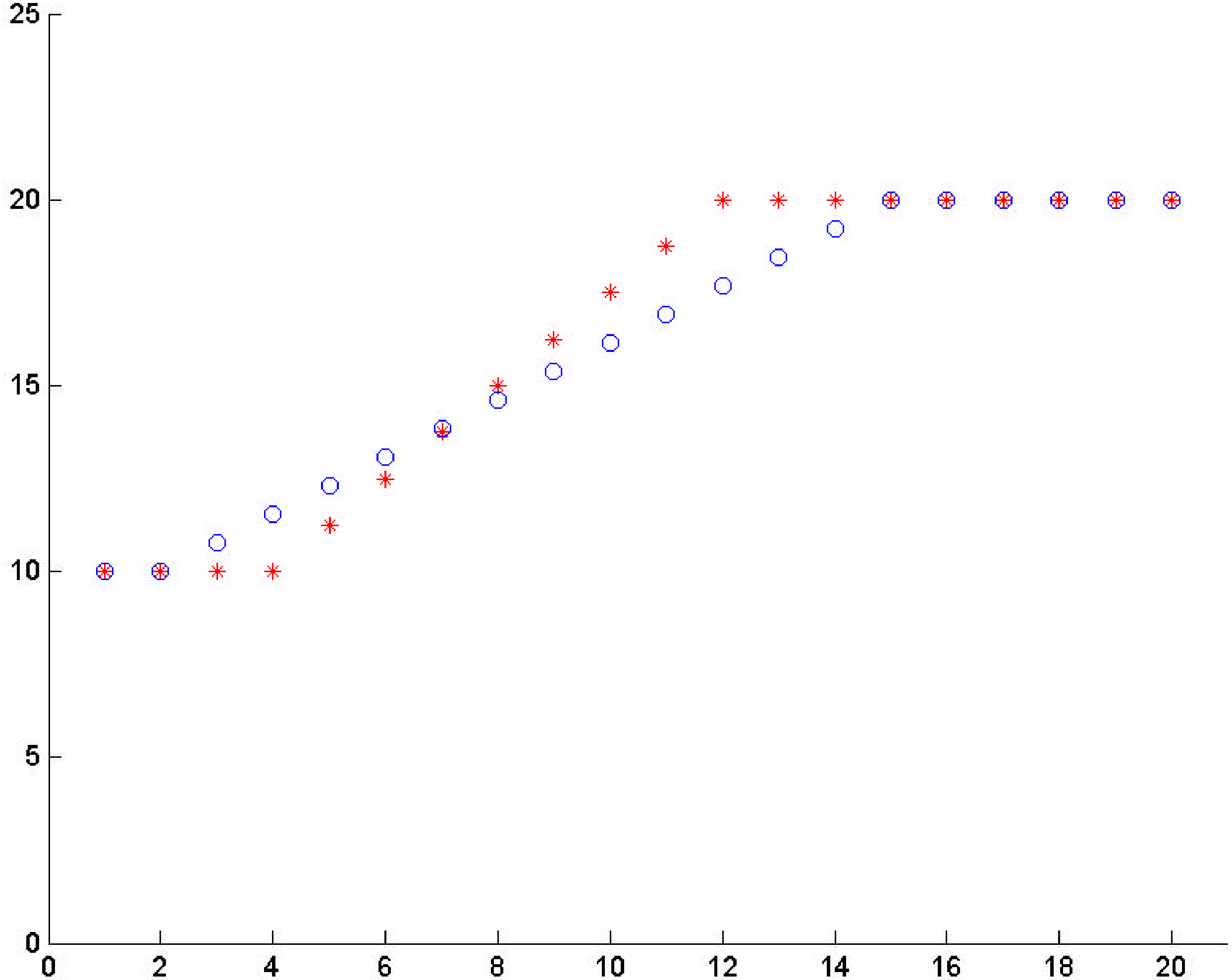
•Two important consequences: (a) it becomes iterative, (b) you need a good approximation, in this case you need to know the correct alignment (shift) to within a “few” pixels

•The 2D case is the exact analog of this 1D tutorial, if you understand this, then the 2D case follows

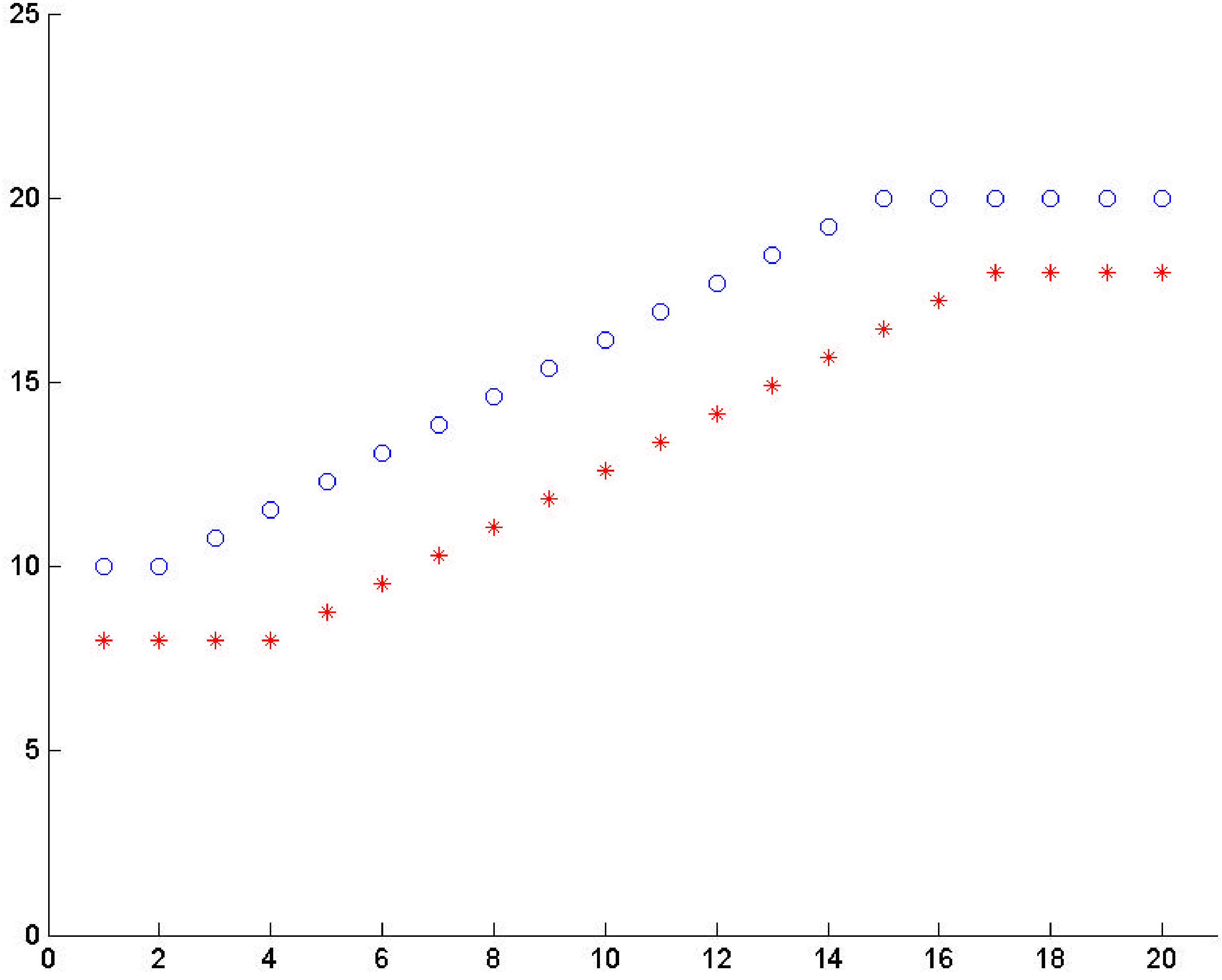
•Because of the above necessity of good *a priori* knowledge of the alignment (shift), this procedure only works for refinement - not coarse matching

•Good news: the precision of final result can be small fraction (0.01) of a pixel

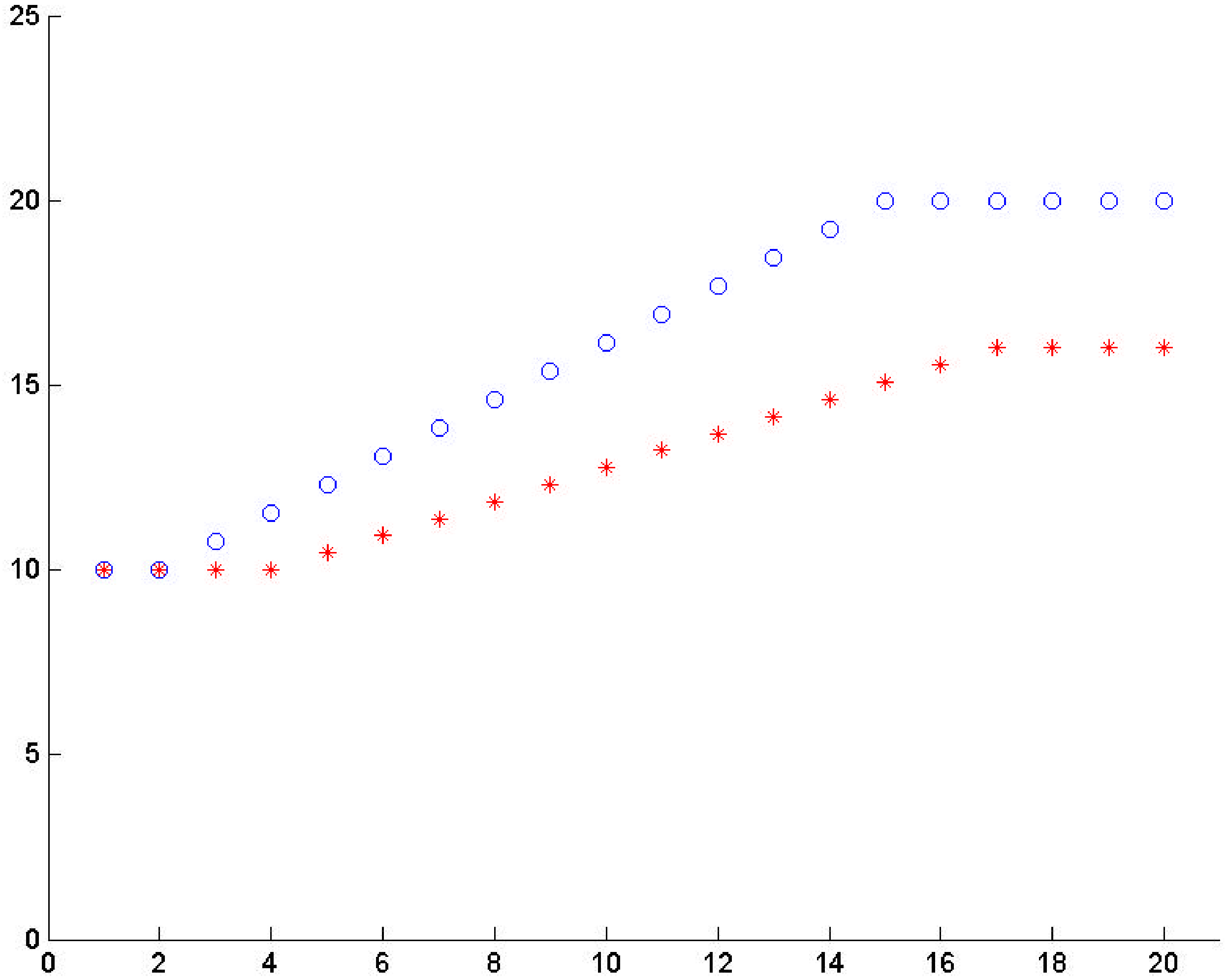
Show Potential Need for Horizontal Scale Parameter



Show Potential Need for Vertical Shift Parameter



Show Potential Need for Vertical Scale Parameter



Condition (Observation) Equation – First Only with Horizontal (Parallax) Shift Parameter

The condition says that, at same location, the left and right intensities are the same

$$u_i = v_i$$

$$u_i - v_i = 0$$

$$F = u_i - v_i = 0$$

Now, introduce a shift parameter

$$F = u_{i+x} - v_i = 0$$

LSM Condition Equation

Linearize the equation using taylor series, at x^0

$$F \approx F^0 + \frac{\partial F}{\partial x}\Delta x = 0$$

$$F \approx \left[u_{i+x^0} - v_i\right] + \frac{\partial F}{\partial x}\Delta x = 0$$

But we always linearize at x = 0, then after estimation we resample so that the alignment gets progressiv ely better and better, and this assumption gets progressiv ely more and more justified

$$F \approx \left[u_i - v_i\right] + \frac{\partial F}{\partial x}\Delta x = 0$$

How de we get derivative ? Get it numericall y!

LSM Condition Equation

$$\frac{\partial F}{\partial x} \approx \frac{\Delta F}{\Delta x} = \frac{(u_{i+\Delta x} - v_i) - (u_i - v_i)}{\Delta x}$$

$$\frac{\partial F}{\partial x} \approx \frac{u_{i+\Delta x} - u_i}{\Delta x}$$

$$\frac{\partial F}{\partial x} \approx \frac{\Delta u}{\Delta x}$$

call it U_x just the slope of u !

LSM Condition Equation

$$F \approx u_i - v_i + U_x \Delta x = 0$$

rearrange,

$$v_i = U_x ? x + u_i$$

add a correction, r_i

$$v_i + r_i = U_{x_i} \Delta x + u_i$$

observation

correction

coefficient

unknown

constant

Least Squares Model

$$r_i - U_{x_i} \Delta x = u_i - v_i$$

same form as LS: $v + B\Delta = f$

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = \begin{bmatrix} -U_{x_1} \\ -U_{x_2} \\ \vdots \\ -U_{x_n} \end{bmatrix} [\Delta x] = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ \vdots \\ u_n - v_n \end{bmatrix}$$

LS solution for indirect observations

$$\Delta = (\mathbf{B}^{\mathbf{T}}\mathbf{W}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{W}\mathbf{f}$$

$\mathbf{W}$ is a weight matrix which may be identity