Image Coordinate Refinement & Camera Calibration

Perspective center

Image (2D) and sensor (3D) coordinate system defined with respect to focal plane, optical axis and reference direction

z

Optical axis

Measurement system (row,column) defined with respect to digital image

y

x

Principal point

Focal plane
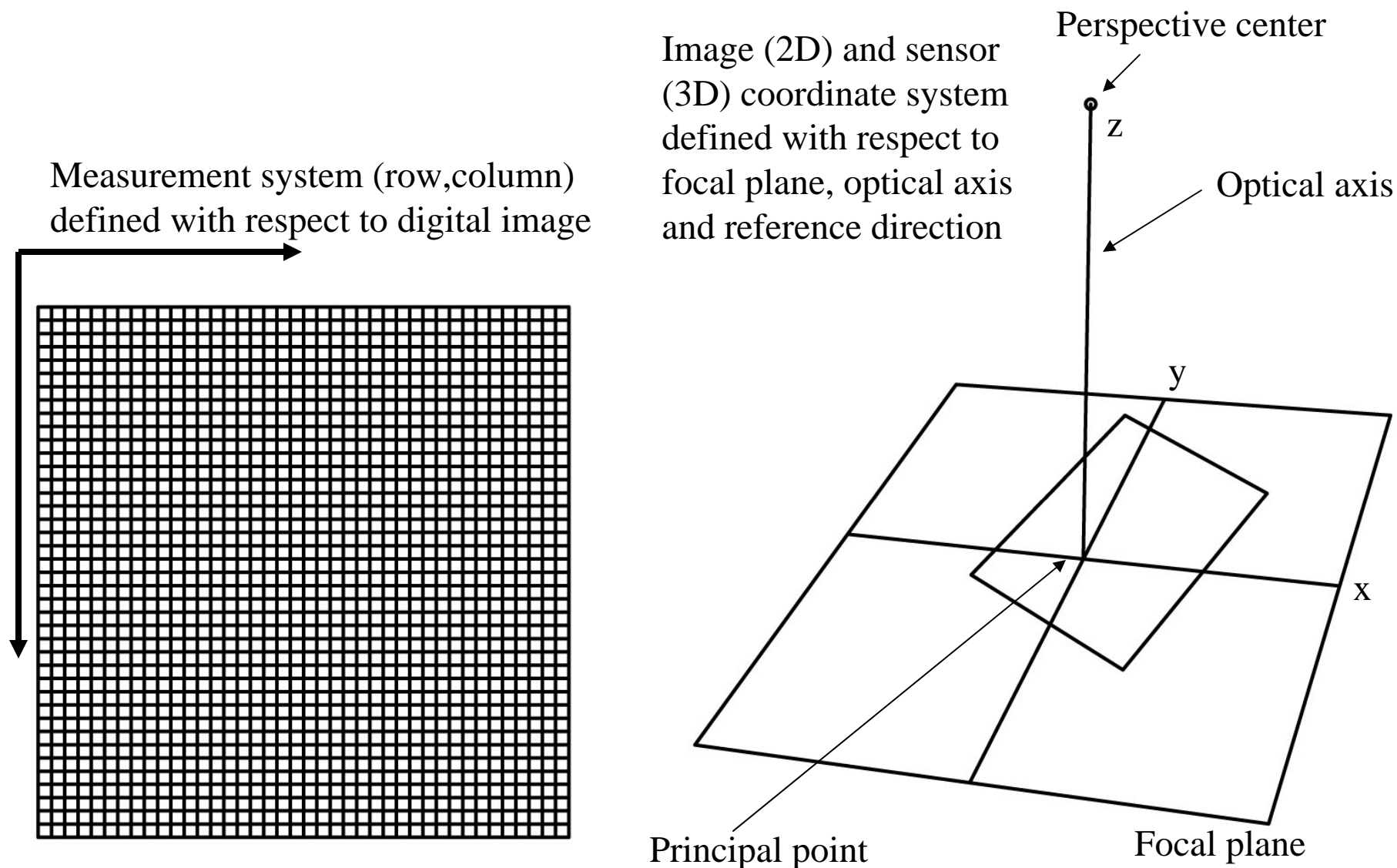
Coordinate Transformations to relate measurement (r,c) and image (x,y) coordinate systems

Write equations at fiducial marks or reseau marks or any other fixed points which are known or observed in *both* systems. Solve for parameters with those equations – then apply at all other measured points.

4 - parameter, nonlinear

$$\begin{bmatrix} r \\ c \end{bmatrix} = \lambda \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

4 - parameter, linear

$$\begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$
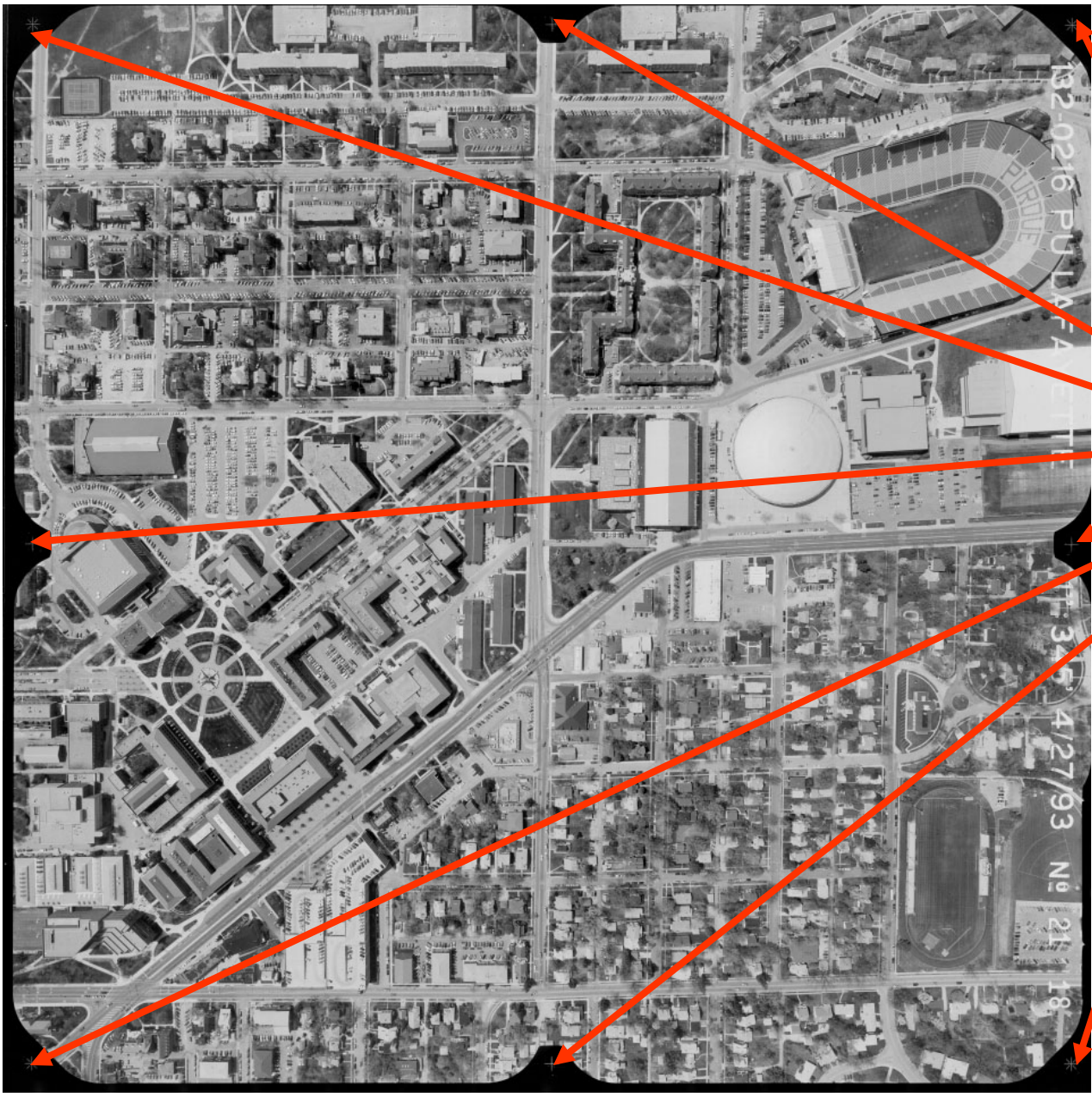
6 - parameter, affine, nonlinear

$$\begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

6 - parameter, affine, linear

$$\begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

8 - parameter, nonlinear

$$r = \frac{a_1 x + b_1 y + c_1}{a_0 x + b_0 y + 1}$$

$$c = \frac{a_2 x + b_2 y + c_2}{a_0 x + b_0 y + 1}$$

Fiducial marks – given in camera calibration report
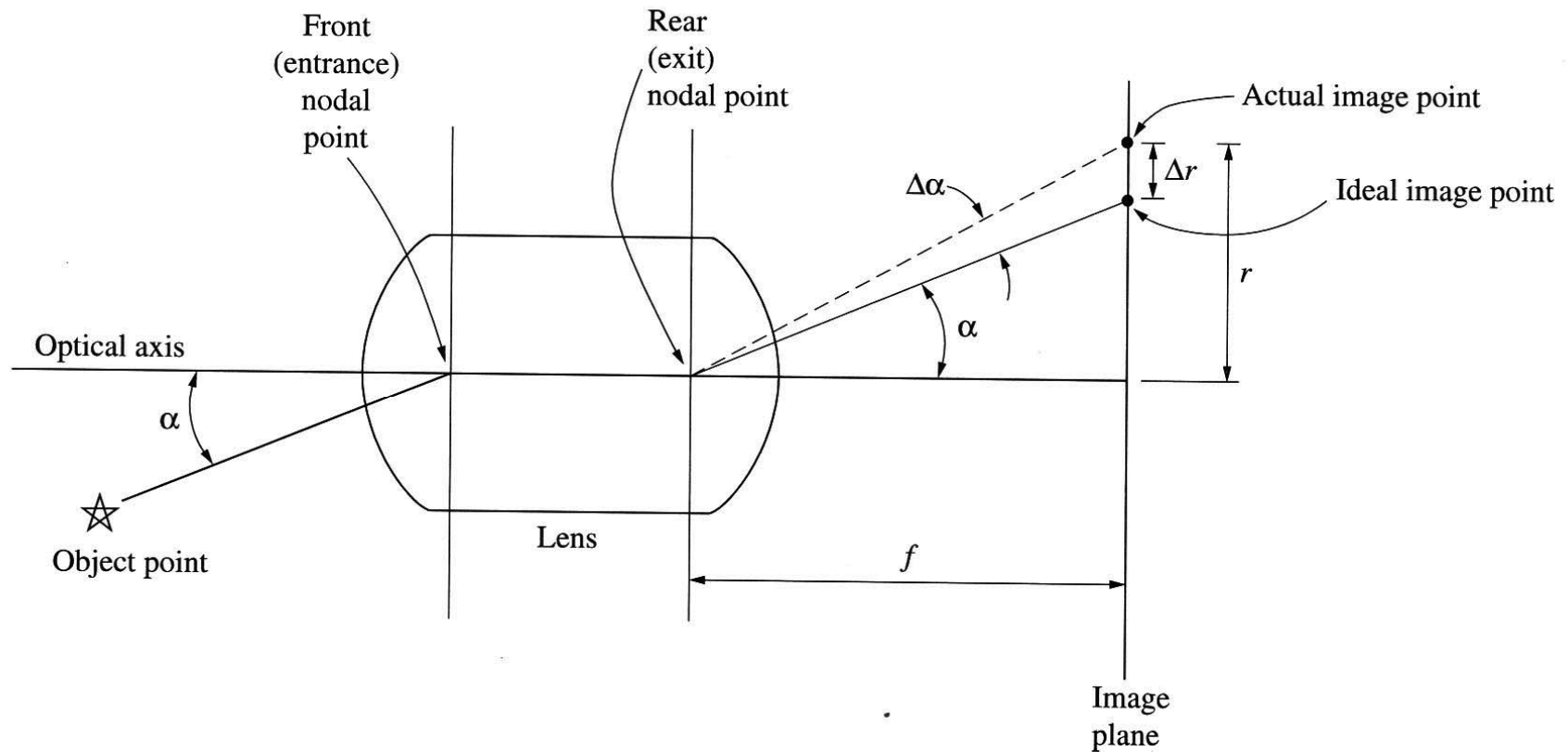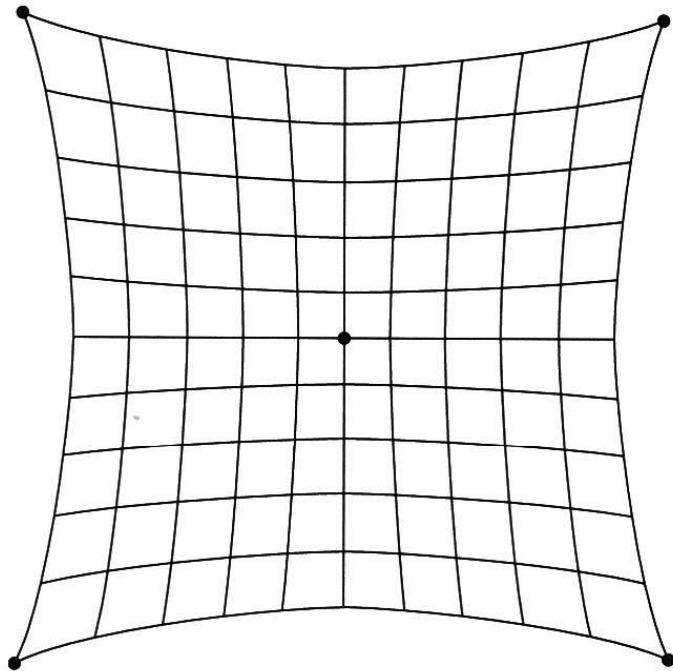
Lens Distortion (radial)

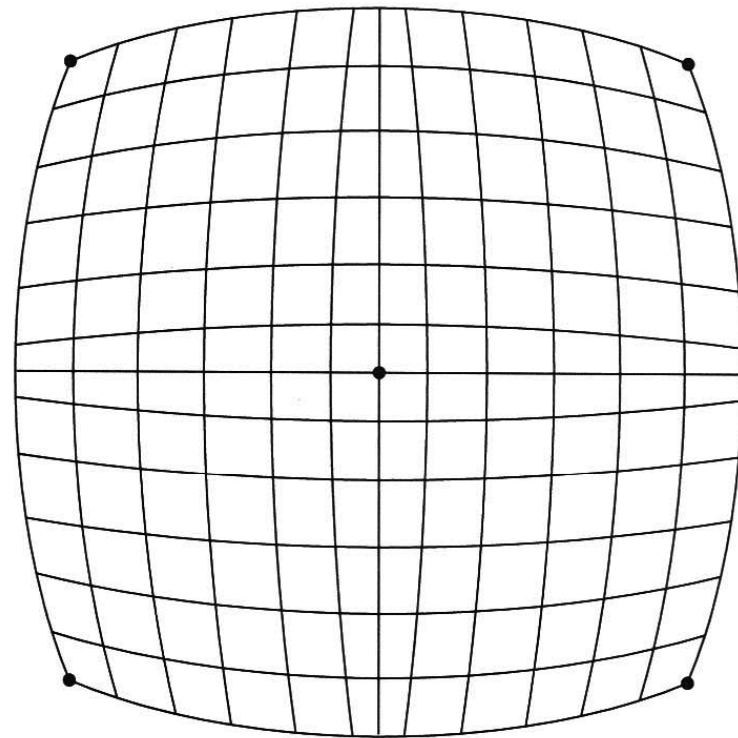**Figure 4-7** Lens distortion.

$$\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + k_3 r^7$$

Exaggerated effects of radial lens distortion

Positive   Negative

Figure 3-7 Positive and negative radial distortion.

Laboratory calibration instrument to project targets at fixed angles into the camera (multi-collimator, USGS laboratory)

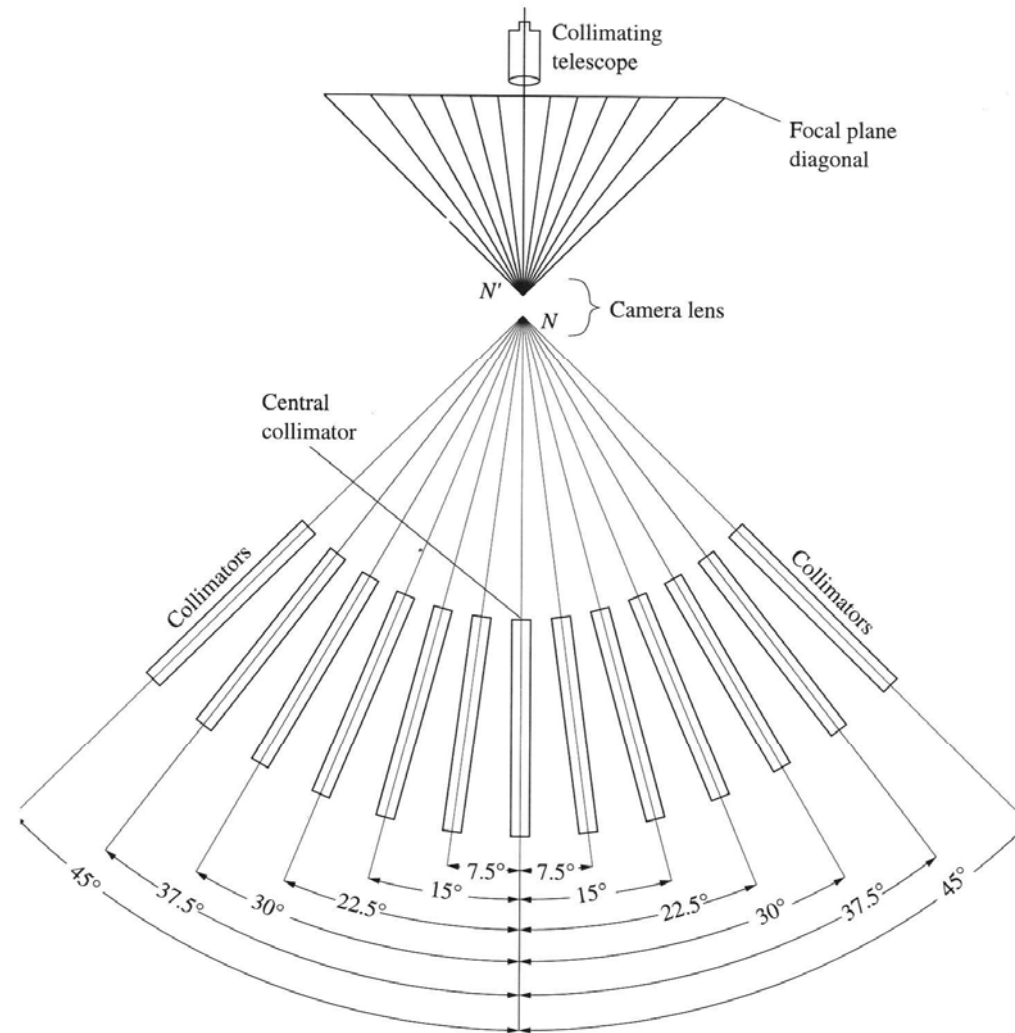**Figure 3-31** Schematic of a multicollimator for mapping camera calibration.

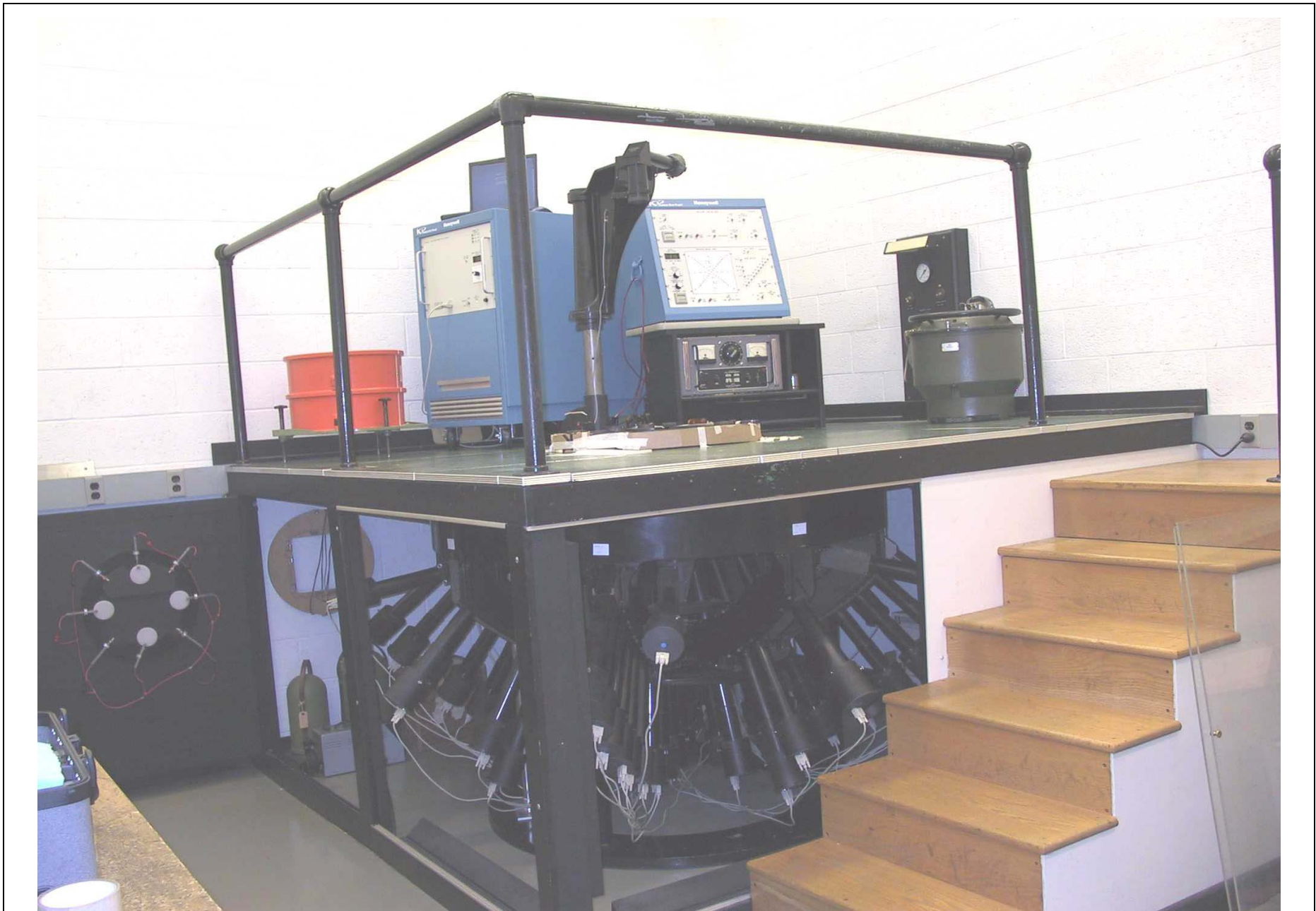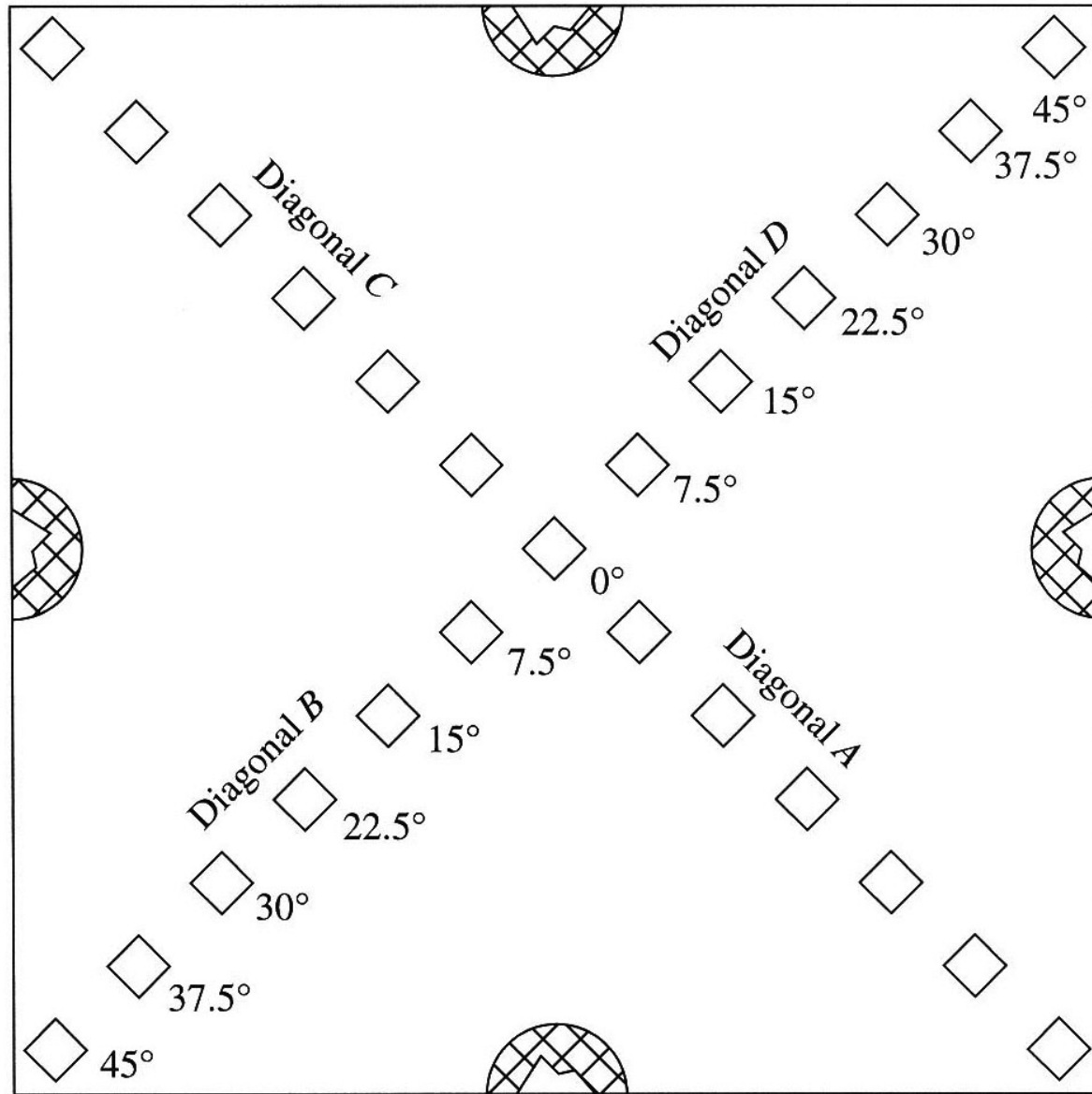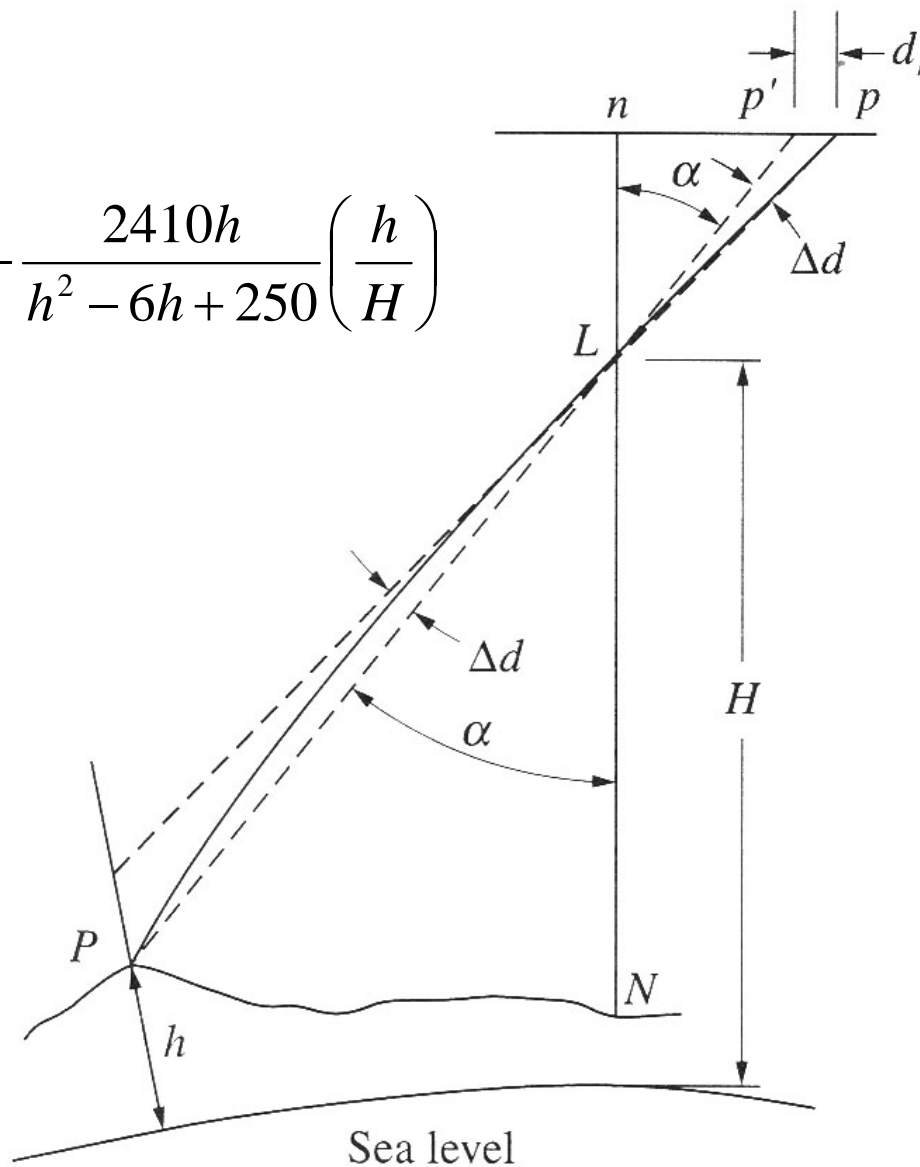CE 503 – Photogrammetry I – Fall 2002 – Purdue University

Image resulting from multi-collimator calibration

Atmospheric Refraction

$$\Delta d = k \tan \alpha$$

$$k = \frac{2410H}{H^2 - 6H + 250} - \frac{2410h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)$$

United States Department of the Interior

U.S. GEOLOGICAL SURVEY
Reston, Virginia 20192

REPORT OF CALIBRATION
of Aerial Mapping Camera

December 18, 1998

Camera type: Wild RC10
Lens type: Wild Universal Aviogon /4
Nominal focal length: 153 mm

Camera serial no.: 1394
Lens serial no.: 13055
Maximum aperture: f/4
Test aperture: f/4

Submitted by: Dickerson Aerial Surveys, Inc.
Lafayette, Indiana

Reference: Letter dated December 14, 1998, from Mr. John D. Dickerson.

These measurements were made on Kodak Micro-flat glass plates, 0.25 inch thick, with spectroscopic emulsion type 157-01 Panchromatic, developed in D-19 at 68° F for 3 minutes with continuous agitation. These photographic plates were exposed on a multicollimator camera calibrator using a white light source rated at approximately 5200K.

I. Calibrated Focal Length: 153.077 mm

II. Lens Distortion

Field angle:	7.5°	15°	22.7°	30°	35°	40°
Symmetric radial (um)	-1	-2	-1	0	2	1
Decentering (um)	0	0	0	1	1	2

Symmetric radial distortion parameters	Decentering distortion parameters	Calibrated principal point
$K_0 = 0.6142 \times 10^{-4}$	$P_1 = -0.1235 \times 10^{-7}$	$x_p = 0.005$ mm
$K_1 = -0.1179 \times 10^{-7}$	$P_2 = 0.9974 \times 10^{-7}$	$y_p = -0.004$ mm
$K_2 = 0.4519 \times 10^{-12}$	$P_3 = 0.0000$	
$K_3 = 0.0000$	$P_4 = 0.0000$	
$K_4 = 0.0000$		

The values and parameters for Calibrated Focal Length (CFL), Symmetric Radial Distortion (K_0,K_1,K_2,K_3,K_4), Decentering Distortion (P_1,P_2,P_3,P_4), and Calibrated Principal Point [point of symmetry] (x_p,y_p) were determined through a least-squares Simultaneous Multiframe Analytical Calibration (SMAC) adjustment. The x and y-coordinate measurements utilized in the adjustment of the above parameters have a standard deviation (σ) of ±3 microns.

III. Lens Resolving Power in cycles/mm

Area-weighted average resolution: 80

Field angle:	0°	7.5°	15°	22.7°	30°	35°	40°
Radial Lines	113	113	80	57	95	95	67
Tangential lines	113	113	80	67	80	80	67

The resolving power is obtained by photographing a series of test bars and examining the resultant image with appropriate magnification to find the spatial frequency of the finest pattern in which the bars can be counted with reasonable confidence. The series of patterns has spatial frequencies from 5 to 268 cycles/mm in a geometric series having a ratio of the 4th root of 2. Radial lines are parallel to a radius from the center of the field, and tangential lines are perpendicular to a radius.

IV. Filter Parallelism

The two surfaces of the Wild No. 7419, the 500 Pan No. 4006, and the 525 No. 7415 filters accompanying this camera are within 10 seconds of being parallel. The 525 filter was used for the calibration.

V. Shutter Calibration

Indicated exposure time	Effective exposure time	Efficiency
1/200	5.50 ms = 1/180 s	81%
1/400	2.63 ms = 1/380 s	81%
1/600	1.75 ms = 1/570 s	81%
1/800	1.31 ms = 1/760 s	81%
1/1000	1.05 ms = 1/950 s	81%

The effective exposure times were determined with the lens at aperture f/4. The method is considered accurate within 3 percent. The technique used is Method I described in American National Standard PH3.48-1972(R1978).

VI. Film Platen

The film platen mounted in Wild RC10 drive unit No. 1394-68 does not depart from a true plane by more than 13 um (0.0005 in).

This camera is equipped with a platen identification marker that will register "68" in the data strip area for each exposure.

VII. Principal Points and Fiducial Coordinates

```
d    3 (90°)        7              2 (180°)
a
t
a          ┌─────────────────────┐
           │ ╲         D        ╱ │
s          │   ╲             ╱    │
t          │     ╲         ╱      │
r    5  A  │       ╲     ╱        │ B  6
i          │         ╳ PPA        │
p          │       ╱     ╲        │
           │     ╱         ╲      │
s          │   ╱             ╲    │
           │ ╱         C        ╲ │
i          └─────────────────────┘
d
e    1 (0°)         8              4 (270°)
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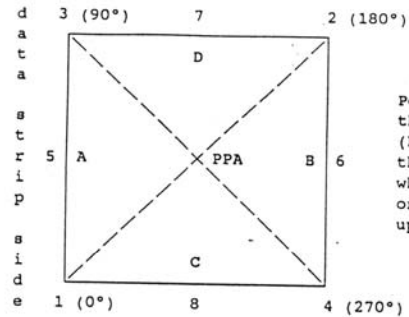
Positions of all points are referenced to the principal point of autocollimation (PPA) as origin. The diagram indicates the orientation of the reference points when the camera is viewed from the back, or a contact positive with the emulsion up. The data strip is to the left.

	X coordinate	Y coordinate
Indicated principal point, corner fiducials	0.003 mm	-0.001 mm
Indicated principal point, midside fiducials	0.004	-0.001
Principal point of autocollimation (PPA)	0.0	0.0
Calibrated principal point (pt. of sym.) x_p, y_p	0.005	-0.004

Fiducial Marks

1	-106.006 mm	-106.003 mm
2	106.003	105.993
3	-105.991	105.999
4	105.998	-106.003
5	-110.002	-0.002
6	110.042	-0.001
7	0.004	109.988
8	0.003	-110.025

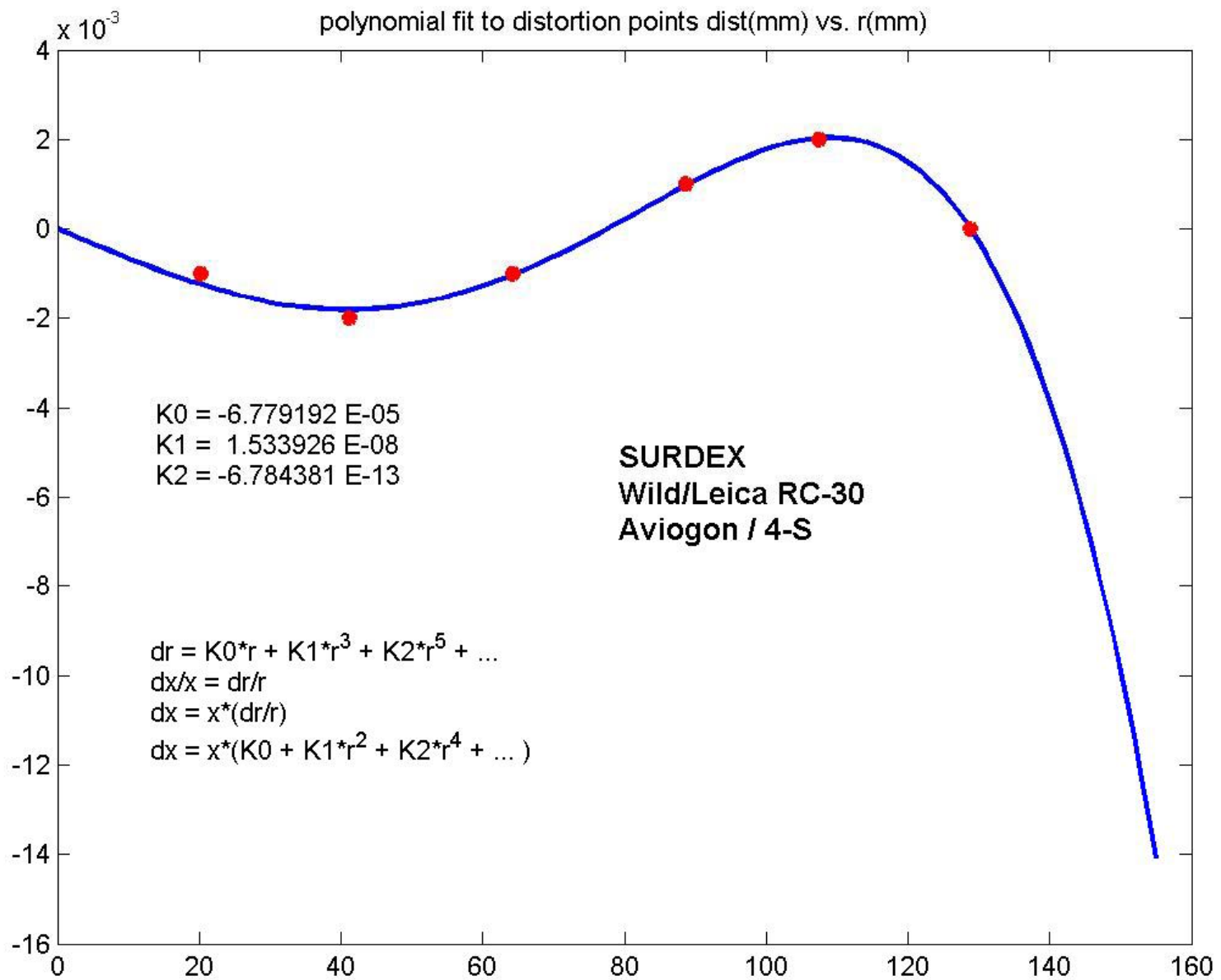VIII. Distances Between Fiducial Marks

Corner fiducials (diagonals)
    1-2: 299.817 mm   3-4: 299.807 mm
Lines joining these markers intersect at an angle of 90° 00' 00"

Midside fiducials
    5-6: 220.044 mm   7-8: 220.013 mm
Lines joining these markers intersect at an angle of 89° 59' 58"
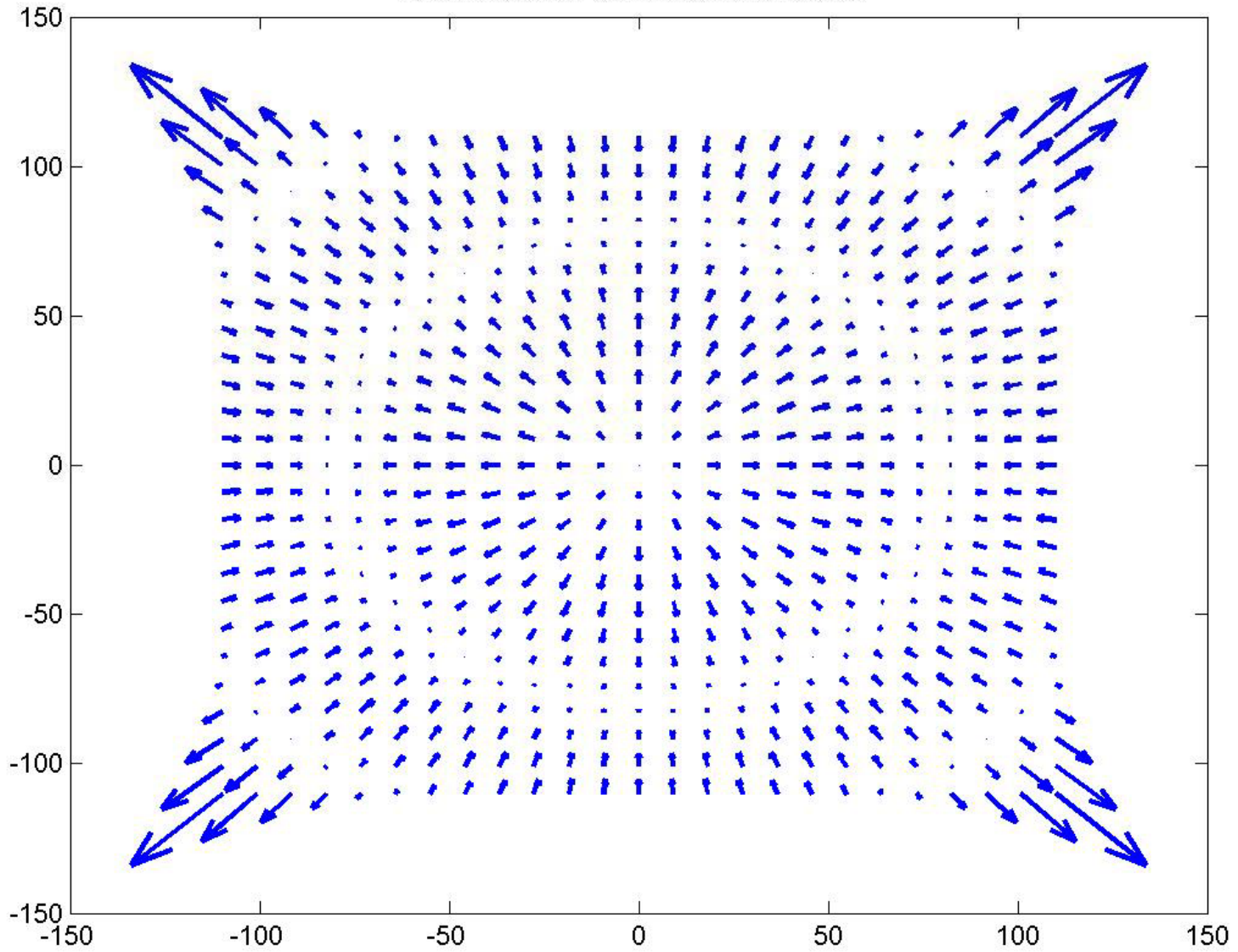
Corner fiducials (perimeter)
    1-3: 212.002 mm   2-3: 211.994 mm
    1-4: 212.004 mm   2-4: 211.996 mm

The method of measuring these distances is considered accurate within 0.003 mm

Note: For GPS applications, the nominal entrance pupil distance from the focal plane is 282 mm.

polynomial fit to distortion points dist(mm) vs. r(mm)

K0 = -6.779192 E-05
K1 = 1.533926 E-08
K2 = -6.784381 E-13

SURDEX
Wild/Leica RC-30
Aviogon / 4-S

$dr = K0*r + K1*r^3 + K2*r^5 + ...$
$dx/x = dr/r$
$dx = x*(dr/r)$
$dx = x*(K0 + K1*r^2 + K2*r^4 + ...)$

radial distortion CORRECTION vectors

decentering distortion CORRECTION vectors

combined distortion CORRECTION vectors