Ce506 – homework #2 solution 6-sep-02

Problem 1.

minimize $f(x, y) = 2x^2 + 3y^2$

subject to $y = 2x + 5$

(a) Solution by substitution

$$f(x) = 2x^2 + 3(2x+5)^2 =$$

$$14x^2 + 60x + 75$$

$$df/dx = 28x + 60 = 0$$

$$x = -2.1429$$

$$y = 0.7143$$

(b) Solution by lagrange multipliers

$$\Phi = 2x^2 + 3y^2 + k(y - 2x - 5)$$

$$\partial\Phi/\partial x = 4x - 2k = 0$$

$$\partial\Phi/\partial y = 6y + k = 0$$

$$\partial\Phi/\partial k = y - 2x - 5 = 0$$

Extract and solve the matrix equation

$$\begin{bmatrix} 4 & 0 & -2 \\ 0 & 6 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ k \end{bmatrix} = \begin{bmatrix} -2.1429 \\ 0.7143 \\ -4.2857 \end{bmatrix}$$

Checks with elimination method

Problem 2 (3-8). Longhand using lagrange mult.

#	obs	Sig	Sig^2	w
a	60-00-00	5 sec	25	9
b	60-00-00	10 sec	100	2.25
c	240-00-25	15 sec	225	1
d	120-00-05	5 sec	25	9

n=4, n_0=2, r=2, weights assume sig_0^2=225

2 condition equations are

$$\hat{a} + \hat{b} = \hat{d}$$

$$\hat{a} + \hat{b} + \hat{c} = 360 \text{ (deg), or}$$

$$v_a + v_b - v_d = 5 \text{ (sec)}$$

$$v_a + v_b + v_c = -25 \text{ (sec)}$$

$$\partial\Phi/\partial v_a = 18 v_a + 2k_1 + 2k_2 = 0$$

$$\partial\Phi/\partial v_b = 2.25 v_b + 2k_1 + 2k_2 = 0$$

$$\partial\Phi/\partial v_c = 2 v_c + 2k_2 = 0$$

$$\partial\Phi/\partial v_c = 18 v_d - 2k_1 = 0$$

$$\partial\Phi/\partial k_1 = 2(v_a + v_b - v_d - 5) = 0$$

$$\partial\Phi/\partial k_2 = 2(v_a + v_b + v_c + 25) = 0$$

Simplify and put into matrix form,

$$\begin{bmatrix} 9 & 0 & 0 & 0 & 1 & 1 \\ 0 & 2.25 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 9 & -1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ -25 \end{bmatrix}$$

$$\Phi' = 9v_a^2 + 2.25 v_b^2 + v_c^2 + 9 v_d^2 + 2k_1(v_a + v_b - v_d - 5) + 2k_2(v_a + v_b + v_c + 25) \rightarrow \min$$

3-8 continued

Solving the prior matrix equation,

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0.3390 \\ 1.3559 \\ -26.6949 \\ -3.3051 \\ -29.7458 \\ 26.6949 \end{bmatrix}$$

Naturally you should round to integer seconds, etc.

You can also solve by elimination, to do so you solve for v's in terms of k's (you can always do this)

$$v_a = -k_1/9 - k_2/9$$
$$v_b = -k_1/2.25 - k_2/2.25$$
$$v_c = -k_2$$
$$v_d = k_1/9$$

Now, plug these into the 2 condition equations,

$$\begin{bmatrix} -0.6667 & -0.5556 \\ -0.5556 & -1.5556 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -25 \end{bmatrix}$$

solving,

$$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -29.7458 \\ 26.6949 \end{bmatrix}$$

Plug these k's into prior equations to obtain the v's – result is same as simultaneous solution.

Problem 3 (3-11) longhand solution with lagrange mult.

#	obs	sig	Sig^2	w
1	20.410	.003	9e-06	9
2	10.100	.003	9e-06	9
3	10.300	.006	36e-06	2.25
4	10.315	.009	81e-06	1

n=4, n_0=2, r=2, weights assume sig_0^2=81e-06,

Need 2 condition equations,

$$\hat{l}_1 - \hat{l}_3 - \hat{l}_2 = 0$$

$$\hat{l}_3 - \hat{l}_4 = 0, \quad \text{or}$$

$$v_1 - v_2 - v_3 = -0.01$$

$$v_3 - v_4 = 0.015$$

$$\partial\Phi/\partial v_1 = 18v_1 + 2k_1 = 0$$

$$\partial\Phi/\partial v_2 = 18v_2 - 2k_1 = 0$$

$$\partial\Phi/\partial v_3 = 4.5v_3 - 2k_1 + 2k_2 = 0$$

$$\partial\Phi/\partial v_4 = 2v_4 - 2k_2 = 0$$

$$\partial\Phi/\partial k_1 = 2(v_1 - v_2 - v_3 + 0.01) = 0$$

$$\partial\Phi/\partial k_2 = 2(v_3 - v_4 - 0.015) = 0$$

Put into matrix form and solve,

$$\begin{bmatrix} 9 & 0 & 0 & 0 & 1 & 0 \\ 0 & 9 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2.25 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.01 \\ 0.015 \end{bmatrix}$$

Solution vector is,

$$\begin{bmatrix} -0.0011 & 0.0011 & 0.0077 & -0.0073 & 0.0102 & -0.0073 \end{bmatrix}^T$$

Can also solve by elimination, as previous prob

$$\Phi' = 9v_1^2 + 9v_2^2 + 2.25v_3^2 + v_4^2 + 2k_1(v_1 - v_2 - v_3 + .01) + 2k_2(v_3 - v_4 - 0.015) \rightarrow \min$$