$$\frac{\partial F_1}{\partial x} = ? \quad F_1 = d_1 - \left[(x-x_1)^2 + (y-y_1)^2\right]^{1/2} = 0$$

analytically :

$$\frac{\partial F_1}{\partial x} = -\tfrac{1}{2}\left[\cdot\right]^{-1/2} \cdot 2(x-x_1)$$

$$\frac{\partial F_1}{\partial y} = -\tfrac{1}{2}\left[\cdot\right]^{-1/2} \cdot 2(y-y_1)$$

evaluate @ current approximation

x_0, y_0

numerical approximation

$$\lim_{\Delta x \to 0} \frac{F_1(x+\Delta x, y) - F_1(x,y)}{\Delta x} = \frac{\partial F_1}{\partial x}$$

$$\frac{F_1(x+\Delta x, y) - F_1(x,y)}{\Delta x} \approx \frac{\partial F_1}{\partial x}$$

$$\frac{\partial F_1}{\partial y} \approx \frac{F_1(x, y+\Delta y) - F_1(x,y)}{\Delta y}$$

Numerical Approach

Dilemma:

Δx too large : poor accuracy

Δx too small : numerical instability

```
F1 = sym('d1-sqrt((x-30)^2 +
  (y-150)^2)');

diff(F1, 'x')

  x = ____;
  y = ____;
  d1 = ____;
B(1,1)= eval(diff(F1, 'x'))
B(1,2)= eval(diff(F1, 'y'))
```

matlab : maple

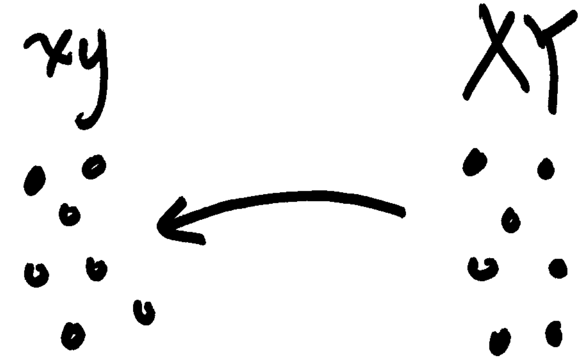
my example : simplest

<u>caution</u> : matlab licensing issues

LS linear problem, similar RPC

$$x = \frac{a_0 + a_1 X + a_2 Y}{1 + c_1 X + c_2 Y}$$

$$y = \frac{b_0 + b_1 X + b_2 Y}{1 + c_1 X + c_2 Y}$$



8-parameter transformation

$a_0, a_1, a_2, b_0, b_1, b_2, c_1, c_2$

$$\left.\begin{aligned} x + x c_1 X + x c_2 Y &= a_0 + a_1 X + a_2 Y \\ y + y c_1 X + y c_2 Y &= b_0 + b_1 X + b_2 Y \end{aligned}\right\} \quad \begin{aligned} & x + v_x \\ & y + v_y \end{aligned}$$

$$v + B\Delta = f$$

$$\underbrace{v_x}_{v} \underbrace{- a_0 - a_1 X - a_2 Y + x c_1 X + x c_2 Y}_{B\Delta} = \underbrace{-x}_{f}$$

$$\underbrace{v_y}_{v} \underbrace{- b_0 - b_1 X - b_2 Y + y c_1 X + y c_2 Y}_{B\Delta} = \underbrace{-y}_{f}$$

$$\begin{bmatrix} v_{x_i} \\ v_{y_i} \end{bmatrix} + \begin{bmatrix} -1 & -X_i & -Y_i & 0 & 0 & 0 & x_i X_i & x_i Y_i \\ 0 & 0 & 0 & -1 & -X_i & -Y_i & y_i X_i & y_i Y_i \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -x_i \\ -y_i \end{bmatrix}$$

1 point $\rightarrow$ 2 equations
4 points $\rightarrow$ 8 equations
5 points $\rightarrow$ 10 equations $\downarrow$ LS

$$\Delta = (B^T W B)^{-1} B^T W f$$

---

example: Nonlinear LS

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ +f \end{pmatrix} = \lambda \left[\underline{\underline{M_a}} M_c M \left(\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \left[\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} + \underline{\underline{\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}}} \right] \right) \right]$$

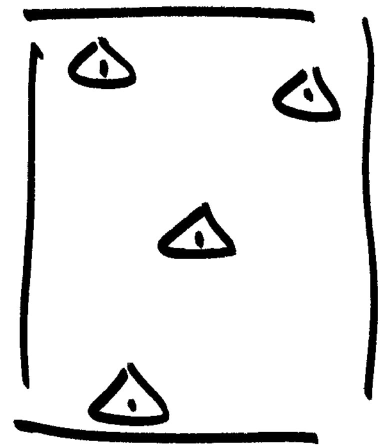
" $\begin{matrix} u \\ v \\ w \end{matrix}$

parameters

$$x = x_0 + f\, u/w$$

$$y = y_0 + f\, v/w$$

$$\begin{cases} F_x = x^{(0)} - x_0 - f\,\dfrac{u}{w} \\ F_y = y - y_0 - f\,\dfrac{v}{w} \end{cases}$$



$$dx = \underline{dx_0} + \underline{dx_1}\,t + \underline{dx_2}\,t^2$$

if 4 GCP's : 2 eqn. / point $\Rightarrow$ 8 eqn.

suppose : carry 6 unknowns

$dx_0, dx_1, dy_0, dy_1, dz_0, dz_1$