Finish quaternion description

<u>A-68</u>

$$M_{\text{axis-angle}} = \begin{bmatrix} \_\_ & \_\_ & \alpha\overset{\cancel{\beta}}{\downarrow}\!\!\!\!\gamma(1-\cos\theta)+\beta\sin\theta \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

correction for page 375
ok, page 449

$$\alpha, \beta, \gamma, \theta \rightarrow q$$

$$q_s = \cos\frac{\theta}{2}$$

$$\begin{pmatrix} q_i \\ q_j \\ q_k \end{pmatrix} = \sin\left(\frac{\theta}{2}\right) \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$q \to \alpha \beta \gamma \theta$

$$\cos\theta = q_s^2 - \left(q_i^2 + q_j^2 + q_k^2\right)$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{\sqrt{q_i^2 + q_j^2 + q_k^2}} \begin{pmatrix} q_i \\ q_j \\ q_k \end{pmatrix}$$

$M \to \alpha, \beta, \gamma, \theta$?

$$\theta = \cos^{-1}\left(\frac{\mathrm{tr}\, M - 1}{2}\right)$$

tr = sum of diagonal elements

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{2 \sin\theta} \begin{pmatrix} m_{32} - m_{23} \\ m_{13} - m_{31} \\ m_{21} - m_{12} \end{pmatrix}$$

singularity with axis/angle parameters when $M = I_3$

rules for multiplication $\quad pq$

$$p = \begin{pmatrix} p_s \\ p_i \\ p_j \\ p_k \end{pmatrix} \qquad q = \begin{pmatrix} q_s \\ q_i \\ q_j \\ q_k \end{pmatrix}$$

$$P = \begin{bmatrix} p_s & -p_i & -p_j & -p_k \\ p_i & p_s & -p_k & p_j \\ p_j & p_k & p_s & -p_i \\ p_k & -p_j & p_i & p_s \end{bmatrix}$$

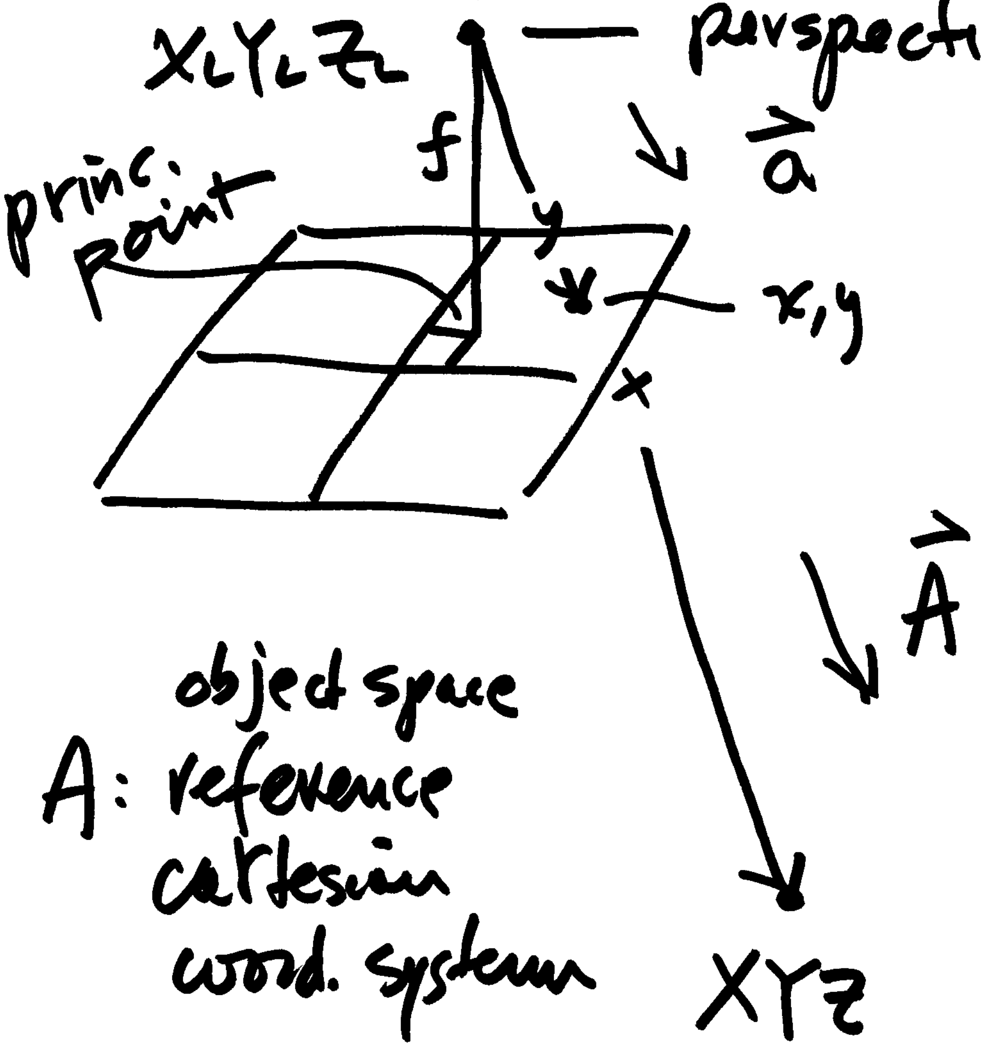
← assumes textbook order of components !

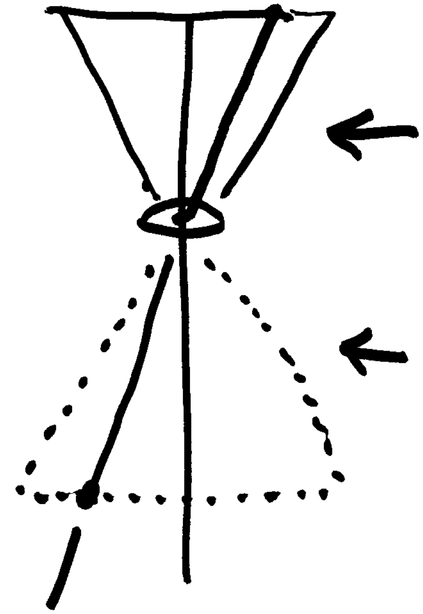
$$pq = Pq$$

Quickbird

<u>textbook</u> $\begin{pmatrix} q_s \\ q_i \\ q_j \\ q_k \end{pmatrix}$, elsewhere $\begin{pmatrix} q_i \\ q_j \\ q_k \\ q_s \end{pmatrix}$

condition equation for QB (Quickbird = Digital Globe)





object space
A: reference cartesian coord. system

a: image space coord. system

$$\vec{a} = \underline{\underline{\lambda}}\,\underline{\underline{M}}\,\vec{A}$$

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda \boxed{M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$= \lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\frac{x - x_0}{-f} = \frac{u}{w}, \quad \frac{y - y_0}{-f} = \frac{v}{w}$$

$$x - x_0 = -f \frac{u}{w}$$

$$y - y_0 = -f \, {}^{v}/_{w}$$

$$F_x : \quad x - x_0 + f \, {}^{u}/_{w} = 0$$

$$F_y : \quad y - y_0 + f \, {}^{v}/_{w} = 0$$

evaluate equation with real numbers

not zero : <u>misclosure</u>

GCP

Simplest QB model

$\phi, \lambda, h \longrightarrow$ ECF cartesian

λ: Neg West. Hem.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{ECF} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ ((1-e^2)N+h)\cdot\sin\phi \end{bmatrix}$$

N: radius curv. Prime Vert

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

WGS84: a: 6378137 (m)

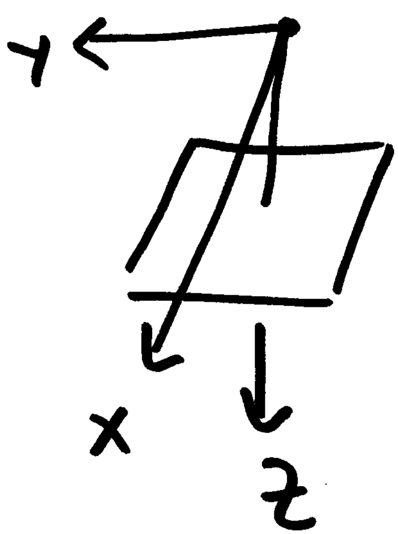
$$f = 1/298.257223563$$

$$e = \sqrt{2f - f^2}$$

QB: cond. eqn.

Quickbird condition equation
(relate object + image coordinates
via physical parameters of camera
+ trajectory)

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ +f \end{pmatrix} = \lambda \underset{(\approx I)}{M_c} M \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{ECF} - \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}_{ECF} \right]$$



Ground Point

instantaneous perspective center

exposure station

How to find $\underline{\begin{matrix} X_c \\ Y_c \\ Z_c \end{matrix}}$, $\underline{M}$, $\underline{M_c}$?

$$\begin{pmatrix} X - x_0 \\ Y - y_0 \\ +f \end{pmatrix} : \begin{pmatrix} 0 - x_0 \\ -S - y_0 \\ +f \end{pmatrix}$$

$$\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix}, \quad M, \quad M_c$$

find these parameters in corresponding files

$\downarrow$ .eph ✓ $\quad$ $\downarrow$ .att ✓ $\quad$ $\downarrow$ .geo ✓

1st line time =

$dt = 1/6900$ sec

$\boxed{t_{line}}$ = time line 1 + line * dt

0, 1, 2, ...

1, 2, 3, ...

(line − 1)