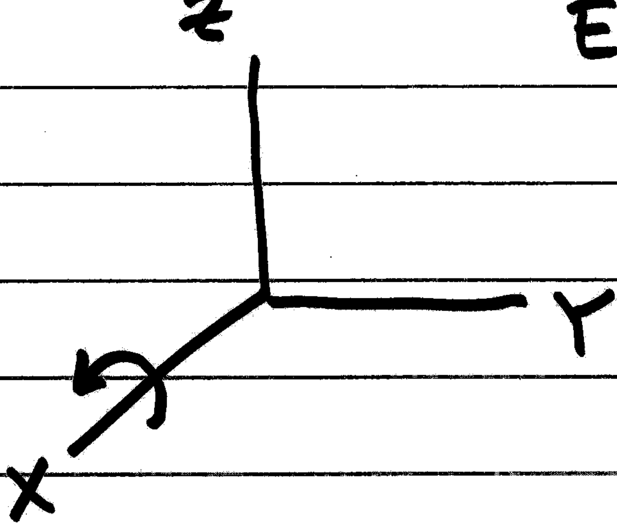
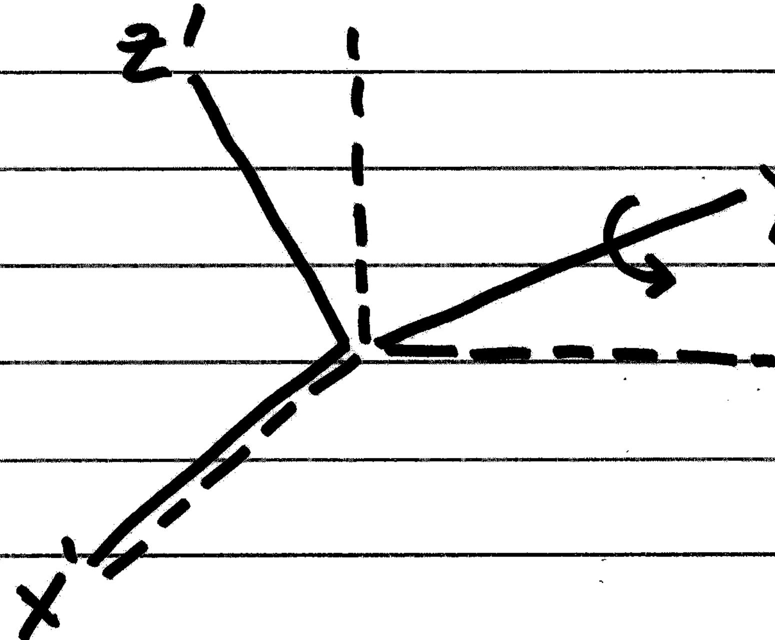
Elementary Rotation Matrices





Sequential rotation

any rotation can be decomposed into product of 3 elementary rotations

$M = M_z M_y M_x$, $R_x R_y R_z$

$R_1 R_2 R_3$

$M = M_z M_y M_x M_z$

given M, can extract factors if

- order specified
- not a critical configuration

gimbal lock

$\underset{3,3}{M}$: multiply out symbollically $M_z M_y M_x$

then solve for $\Theta_x, \Theta_y, \Theta_z$

$\Theta_x : \omega$, $\Theta_y : \phi$, $\Theta_z : k$

aero: roll, pitch, yaw

Euler Angles

Azimuth, Elevation, view axis (terrestrial)

$$M_x(\omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$$

$$M_y(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$M_z(\kappa) = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

interpretation of Seq. Rotations

→ rotating axes (Right Hand Rule)
→ object stationary

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if you want object to rotat
+ axes stationary
⇒ use transpose

matlab implementation

m1, m3

```
% m2.m
function m = m2(phi)
m= [cos(phi) 0 -sin(phi); 0 1 0;
    sin(phi) 0 cos(phi)];
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```
m= m3(kappa)* m2(phi)*
    m1(omega);
```

$$\underset{3,1}{Y} = \underset{3,3}{M}\,\underset{3,1}{X} = Y = \underbrace{M_3(k)\,\underbrace{M_2(\Phi)\,\underbrace{M_1(\omega)\cdot X}_{\text{primary}}}_{\text{secondary}}}_{\text{tertiary}}$$

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conversion ECI ⟷ ECF

ECI ″ ICRS, ECF ″ ITRS

use IAU 1980
nutation theory

start with <u>UTC</u> of event

$X \quad \dot{X}$
$Y \quad \dot{Y}$
$Z \quad \dot{Z}$

Bulletin B : $\begin{cases} UT1 - UTC \\ X_p \quad \text{arc seconds} \ " \\ Y_p \quad \quad " \quad \quad \quad " \end{cases}$

Bulletin C UTC − TAI (−33s)

calendar date Y, M, D

UTC → TT

$$TT = TAI + 32.184\,\mathrm{s} \quad \text{(chart)}$$

$$\underline{\underline{TT}} = \underbrace{UTC}_{\text{obs}} - \underbrace{(UTC - TAI)}_{c} + 32.184\,\mathrm{s}$$

terrestrial time (= ET)

Y, M, D

$$D_f = D + \frac{TT}{\underset{\underset{(86{,}400\,\mathrm{s})}{\shortparallel}}{24\,\mathrm{h}}} \quad \text{(units)}$$

$Y, M, D_f \longrightarrow JD$: julian day

Julian Century since (J2000)

$$T = (JD - 2451545)/36525$$

$$X_{ECF} = \Pi\, \Theta\, N\, P\, X_{ECI} \qquad X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Π: polar motion ($\sim I$)

Θ: earth rotation

N: nutation ($\sim I$)

P: precession ($\sim I$)

$$\zeta = 2306''.2181\,T + .30''188\,T^2 + .017998\,T^3$$

$$Z = 2306.2181\,T + 1.''09468\,T^2 + .018''213\,T^3$$

$$\theta = 2004''.3109\,T - .42''665\,T^2 - .041''833\,T^3$$

$$\underset{(3,3)}{P} = R_z(-Z) \cdot R_y(\theta) \cdot R_z(-\zeta)$$

'' arc seconds

s time seconds

sec/radian : $\left(180/\pi\right) \cdot 3600$

nutation N

inclination of ecliptic

$$\epsilon = 23^\circ.43929111 - 46''.8150\,T - .000''59\,T^2 + .001813\,T^3$$

$$N = R_x(-\epsilon - \underset{=}{\Delta\epsilon})\,R_z(-\underset{=}{\Delta\psi})\,R_x(\epsilon)$$

$$\Delta\epsilon = \sum_{i=1}^{106} (\Delta\epsilon)_i \cos\phi_i$$

$$\Delta\psi = \sum_{i=1}^{106} (\Delta\psi)_i \sin\phi_i$$

$$l = 134^\circ 57' 46''.733 + 477198^\circ 52' 02''.633\,T + 31''.310\,T^2 + .''064\,T^3$$

$$l' = 357^\circ 31' 39''.804 + 35999^\circ 03' 01''.224\,T - .''577\,T^2 - .''012\,T^3$$

$$F = 93^\circ 16' 18''.877 + 483202^\circ 01' 03''.137\,T - 13''.257\,T^2 + .''011\,T^3$$

$$D = 297^\circ 51' 01''.307 + 445267^\circ 06' 41''.328\,T - 6''.891\,T^2 + .''019\,T^3$$

$$\Omega = 125^\circ 02' 40''.280 - 1934^\circ 08' 10''.539\,T + 7''.455\,T^2 + .''008\,T^3$$

$$\phi_i = p_{l,i}\, l + p_{l',i}\, l' + p_{F,i}\, F + p_{D,i}\, D +$$

$$p_{\Omega,i} \quad \Omega$$

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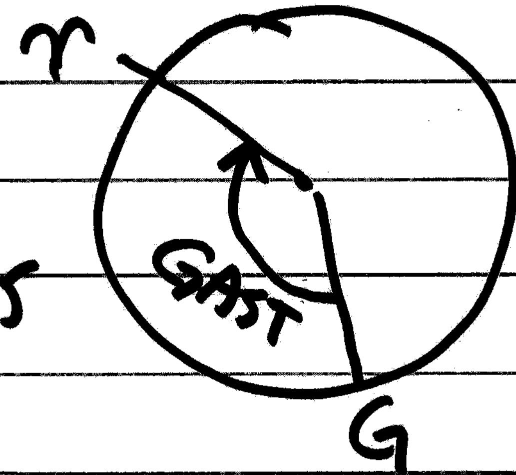
series series Series Series Series

IT ONP Earth Rotation

GAST : greenwich apparent sidereal time



$T_u = \dfrac{(JD(@0h) - 2451545)}{36525}$

integer

$\_\_\_.5$

$GMST(0h\ UT1) = 24110^s.54841 + 8640184^s.812866\,T_u + .093104\,T_u^2 - .0000062\,T_u^3$

$$GMST = GMST(0h) + \underline{\underline{r}}\; UT1$$

$$UT1 = UTC + (UT1 - UTC)$$

typo: extra digit

Bull B.

$$r = 1.0027737\;909\;350\;795 + 5.9006 \times 10^{-11} \cdot T_u - 5.9 \times 10^{-15}\, T_u^2$$

$$GAST = GMST + \Delta\psi \cos\epsilon$$

$$tsec/rad = \frac{86400}{2\pi}$$

$$\Theta = R_z(GAST), \quad m_3(GAST)$$

$\Pi \; \Theta \; N \; P$

polar motion $\quad x_p'' , \; y_p''$

$$\Pi = R_y(-x_p) \cdot R_x(-y_p) \approx I$$

$$\approx \begin{bmatrix} 1 & 0 & +x_p \\ 0 & 1 & -y_p \\ -x_p & +y_p & 1 \end{bmatrix}$$