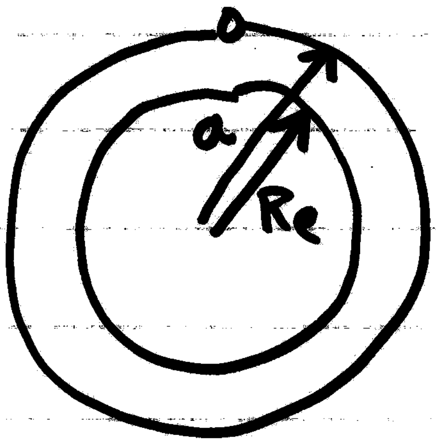
<u>synchronous mode</u> (nadir looking)

mean motion $n = \sqrt{\dfrac{\mu}{a^3}}$ radians/sec

for circular orbit



$R_e = 6367$ km

$H = 400$ km

$a = 6367 + 400 = 6767$ km

$n = 1.1342 \times 10^{-3}$ rad/sec

period $\underline{\underline{P}} = \dfrac{2\pi}{n} = 5539.9$ sec

(92.3 min)

[Clarification of Homework #1]

$$V_g = \frac{2\pi R_e}{P} = 7221 \text{ m/s}$$

desired GSD 0.25 m

$\left.\right\}$ ~ 29,000 lines / second

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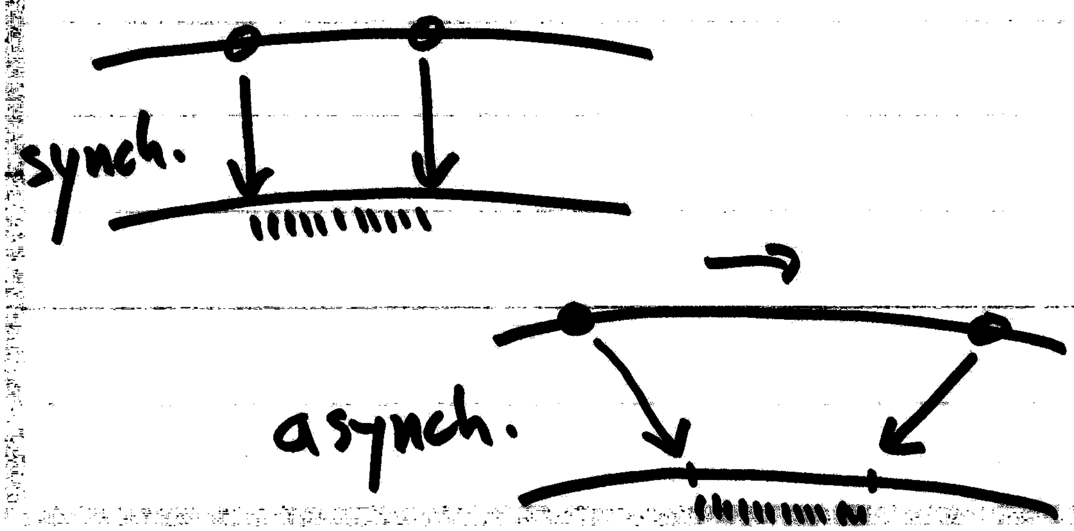
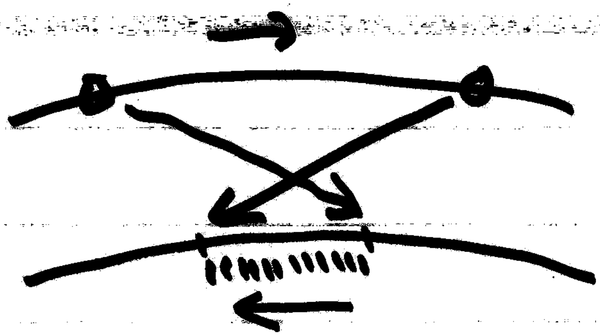
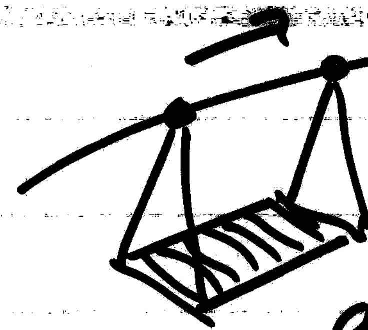
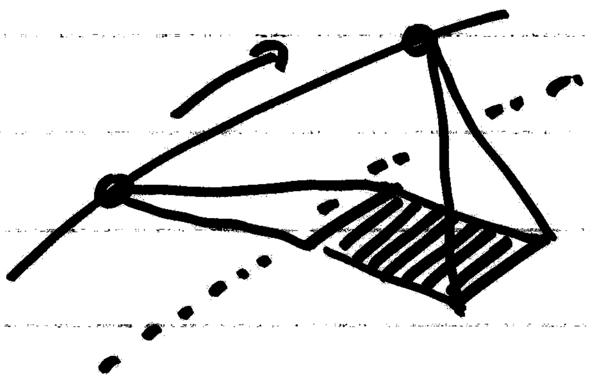
asynchronous mode : actively control the sweep rate by rotating or slewing the camera : obtain any rate you wish.



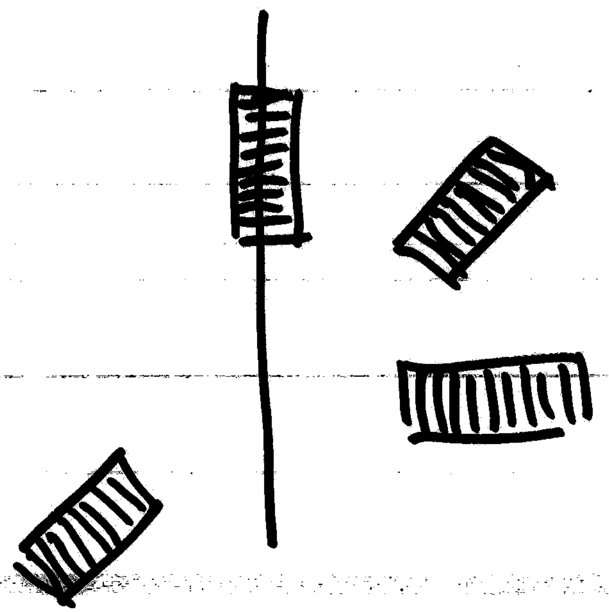




along track



cross track



allow max 10 seconds to acquire scene

24,000 lines, @ 0.25 m/line

6000 m / 10 sec, $\boxed{600\ m/s}$ – V_g

$$T_e < \frac{S}{2V_g}$$

TDI : time delay + integration

$T_{ee} = 32 \times T_e$

H & R_a : choose smallest R_a

other issues

- low H : more atmosphere, orbit will degrade faster
  use fuel ~~faster~~ @ faster rate

- low H : max tilt, limit coverage

on web site : typeset version of derivation

$$\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{1}{r^3}\left(r^2\dot{\vec{r}} - (\vec{r}\cdot\dot{\vec{r}})\,\vec{r}\right)$$

$$\frac{d}{dt}\left(\dot{\vec{r}} \times \vec{h}\right) = \frac{\mu}{r^3}\left(r^2\dot{\vec{r}} - (\vec{r}\cdot\dot{\vec{r}})\,\vec{r}\right)$$

$$\frac{d}{dt}\left(\dot{\vec{r}} \times \vec{h}\right) = \mu\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\frac{d}{dt}\left(\dot{\vec{r}} \times \vec{h}\right) - \mu\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = 0$$

[continuation of lecture #6 on derivation of trajectory equation for the 2-body problem]

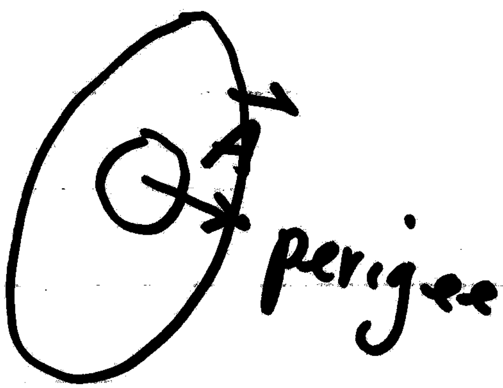
$$\frac{d}{dt}\Big(\underbrace{\dot{\vec r}\times\vec h - \mu\frac{\vec r}{r}}_{\vec A}\Big) = 0$$

$$\boxed{\vec A = \dot{\vec r}\times\vec h - \mu\frac{\vec r}{r}}$$

Runge Lenz vector

Laplace-R-L vector

time derivative $= 0$, $\vec A =$ <u>constant</u>



Vector $\vec A$ points to the perigee

$$\vec{A}\cdot\vec{r} = Ar\cos\theta$$

$$Ar\cos\theta = \Big(\dot{\vec{r}}\times\underbrace{\vec{h}}_{} - \mu\frac{\vec{r}}{r}\Big)\cdot\vec{r}$$

$$\vec{h} = \vec{r}\times\dot{\vec{r}}$$

$$= \Big(\dot{\vec{r}}\times(\vec{r}\times\dot{\vec{r}}) - \mu\frac{\vec{r}}{r}\Big)\cdot\vec{r}$$

$$\vec{r}\cdot\vec{r} = r^2$$

$$= \vec{r}\cdot\big(\dot{\vec{r}}\times(\vec{r}\times\dot{\vec{r}})\big) - (\vec{r}\cdot\vec{r})\frac{\mu}{r}$$

$$= \vec{r}\cdot\underbrace{\big(\dot{\vec{r}}\times(\vec{r}\times\dot{\vec{r}})\big)}_{} - \mu r$$

vector identity #2

$$\vec{r}\cdot\left((\dot{\vec{r}}\cdot\dot{\vec{r}})\vec{r}-(\dot{\vec{r}}\cdot\vec{r})\dot{\vec{r}}\right)-\mu r$$

$$\underbrace{(\vec{r}\cdot\vec{r})(\dot{\vec{r}}\cdot\dot{\vec{r}})-(\vec{r}\cdot\dot{\vec{r}})(\dot{\vec{r}}\cdot\vec{r})}_{\text{vector identity \#3}}-\mu r$$

$$\underbrace{(\vec{r}\times\dot{\vec{r}})}_{\vec{h}}\cdot\underbrace{(\vec{r}\times\dot{\vec{r}})}_{\vec{h}}-\mu r$$

$$=\underbrace{\vec{h}\cdot\vec{h}}-\mu r$$

$$\underline{Ar\cos\theta=h^2-\mu r}$$

$$A\underline{r}\cos\theta + \mu \underline{r} = h^2$$

$$r(A\cos\theta + \mu) = h^2$$

* mistake in notes

$$r = \frac{h^2}{\mu + A\cos\theta}$$

divide by μ

$$r = \frac{h^2/\mu}{1 + A/\mu \cos\theta}$$

$h^2/\mu = P$

$A/\mu = e$

$$\boxed{r = \frac{p}{1 + e\cos\Theta}}$$

equation of a conic section
polar coordinates

e: eccentricity

Θ: true anomaly f, ν

to describe satellite orbit

- conic section
- orientation of plane
- orientation of curve within plane
- time

<u>State vector</u>	<u>Kepler parameters</u>
$\begin{bmatrix} X \\ Y \\ Z \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}$ $\longleftrightarrow$	a, e Ω, i ω f, E, M
t	t

XYZ: ECI
earth centered
inertial coord. sys.

Kepler parameters

Ω : right ascension of ascending node

i : inclination

ω : argument of perigee

a : semi major axis

e : eccentricity

f or ν : true anomaly

(t)

$$\dot{\Omega} = -9.95\left(\frac{R_e}{\underline{\underline{r}}}\right)^{3.5} \cos \underline{\underline{i}} = 0.9856^{\circ} \text{ degrees/day}$$

(assuming circular orbit)

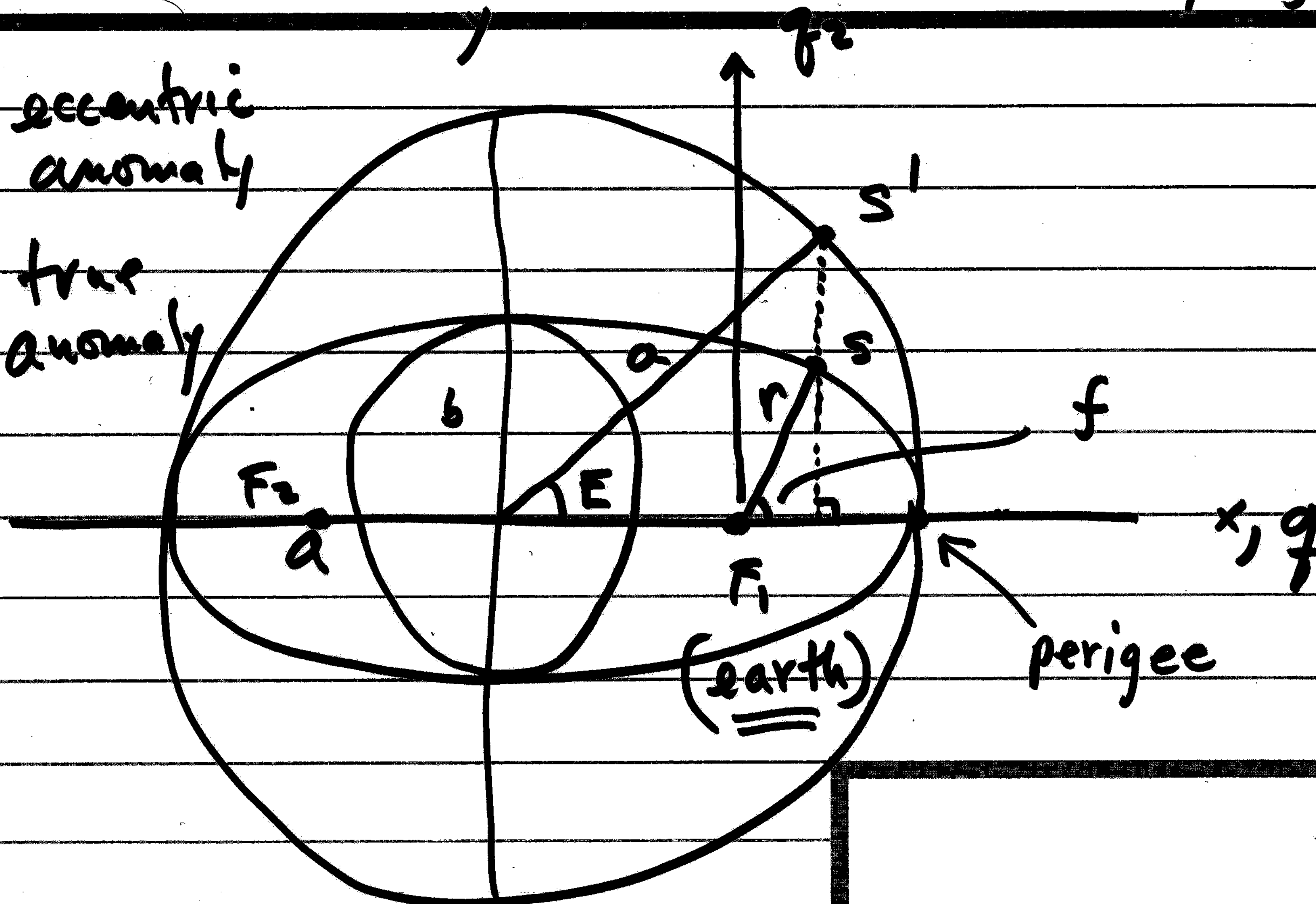
as you go higher : inclination increases

The above equation must be fulfilled so that the orbit precession rate matches earth ~~rotation~~ revolution rate around the sun to yield :

"Sun-Synchronous Orbit"

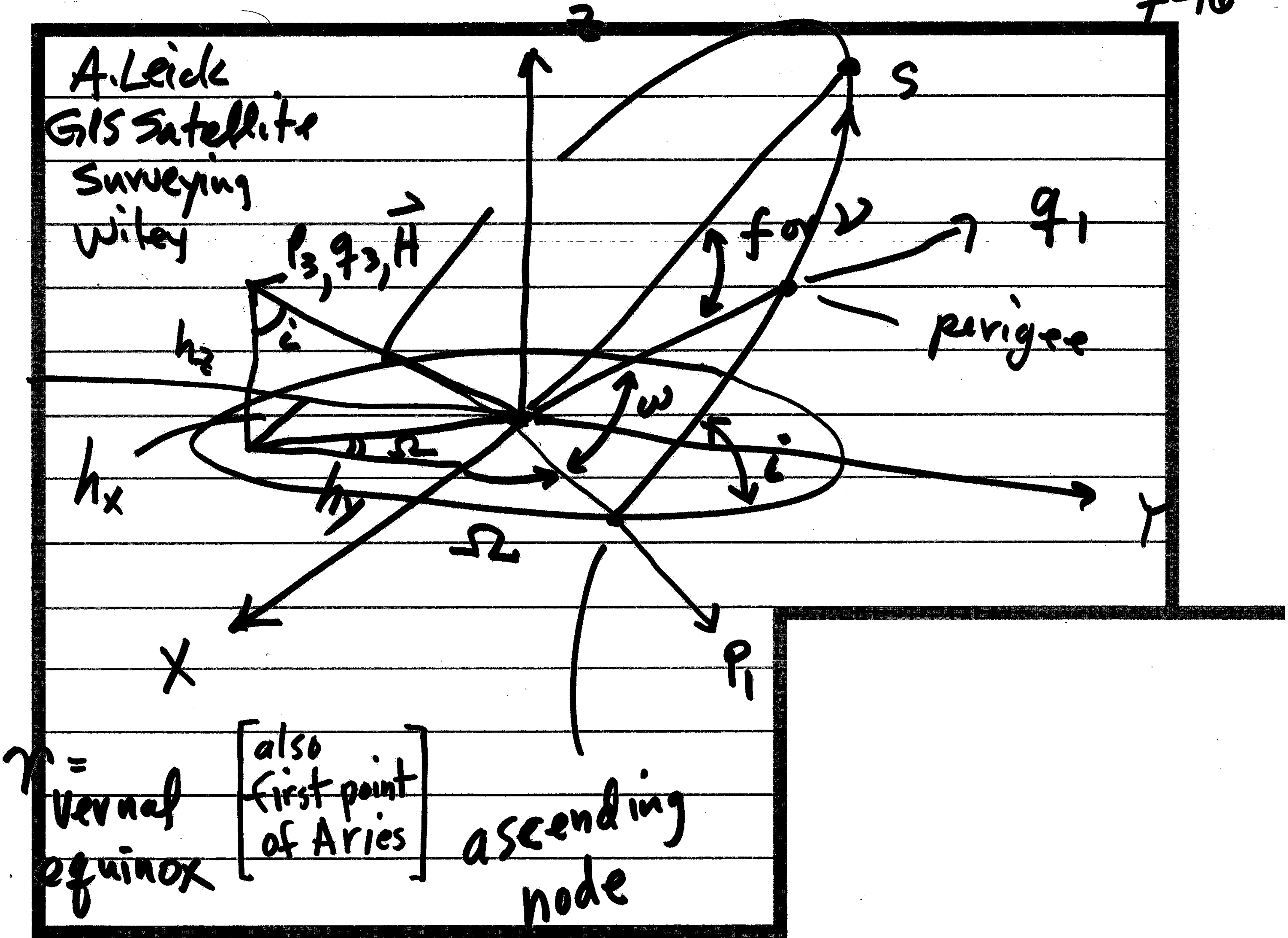
E: eccentric anomaly

f: true anomaly



M: mean anomaly proceeds linearly with time

S: $(a\cos E,\ b\sin E)$


A. Leick
GPS Satellite Surveying
Wiley
z
S
f or ν
q1
perigee
p3, q3, H→
hz
i
hx
hy
Ω
ω
i
Y
X
Ω
p1
♈ = vernal equinox
[also first point of Aries]
ascending node

State vector, Kepler elements

We need to be able to transform from one representation to the other