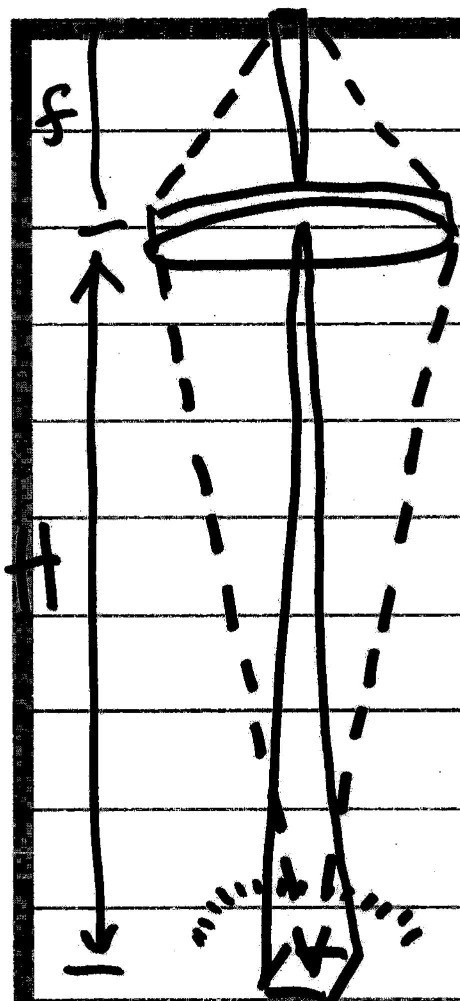
$I = \frac{\Phi}{\omega} = \frac{\Phi_{total}}{4\pi \text{ sr}}$, $4\pi I = \Phi_{total}$

Irradiance @ surface $E = \Phi / A$

$E = \frac{4\pi I}{4\pi d^2}$ ← area of sphere with radius = d

$$E = I / d^2$$



$A_a = \pi R_a^2$

Radiance

$L = \frac{\Phi}{\omega A}$

footprint of single detector element ($\theta = 0$)

s

s

$$L = \frac{\Phi}{\omega A} = \frac{\Phi}{\omega s^2}$$

(circled: $\frac{\Phi}{\omega}$ " I)

$$I = L s^2$$

$$E = I / H^2$$

$$E = L s^2 / H^2$$

$$E = \Phi / A_a$$

$$\frac{\Phi}{A_a} = L\,S^2/H^2$$

$$\Phi = \pi R_a^2\, L\, S^2/H^2$$

$$\Phi = Q/t \quad \text{energy/time}$$

$$Q/t = \pi R_a^2\, L\, S^2/H^2$$

$$Q = t \pi R_a^2 L S^2 / H^2$$

design equation

- Q: energy delivered to aperture by a pixel area during time interval t
- t: exp time, integr time
- R_a: aperture radius
- L: radiance of scene
- S: GSD pixel size on ground
- H: altitude

units: watts, meters, seconds, joules, sr loss factors

- transit through atmosphere
- quantum efficiency
  % of incident photons, which free an electron 20%
- charge transfer

Relate energy Q to photon/electron count

$Q = \#photons \times h\nu$

$h c / \lambda$

h: 6.626×10^{-34} Js planck's c.

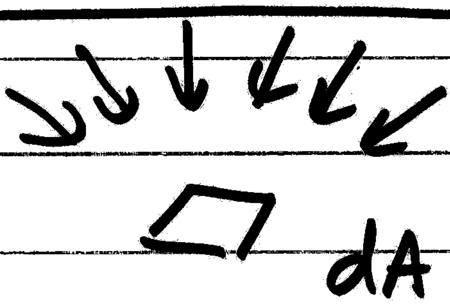
ν: frequency

c: speed of light 3×10^{8} m/s

λ : 550 nm = 550×10^{-9} m
(green)

Q : hc/λ : 3.614×10^{-19} J / green photon

$M = \pi L$
$E = \pi L$



assume: radiance independent of direction

ISOTROPIC

incident power on dA is

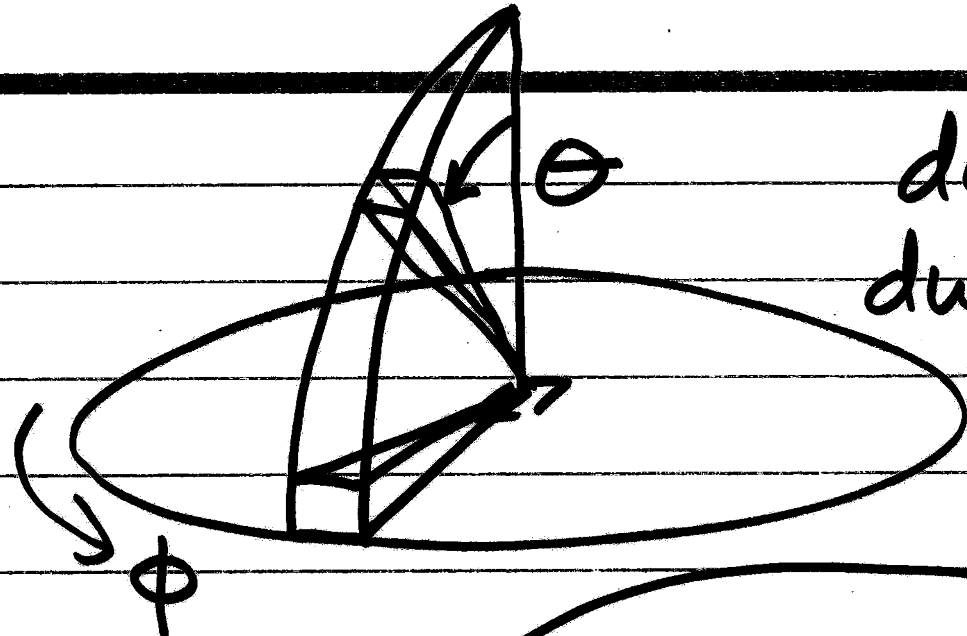
(1) proportional to dA (apparent dA)

(2) proportional to $d\omega$ (solid angle)

(3) proportional to strength of EMR : L
radiance

$$dP = dA \cos\theta \; d\omega \; L$$

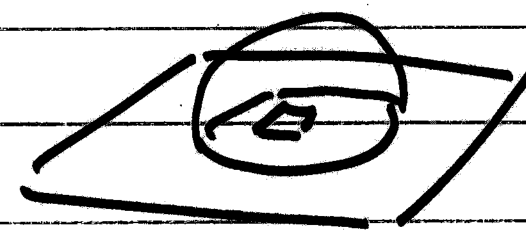
(1) (2) (3)



$d\omega : d\theta\, d\phi$ (horiz)

$d\omega : d\theta\, d\phi \sin\theta$

$dP = dA \cos\theta\, d\omega\, L$



$dP = dA \cos\theta\, d\theta\, d\phi \sin\theta\, L$

want power per unit Area

(dA = 1)

$dE = L \cos\theta \sin\theta\, d\phi\, d\theta$

for total E, integrate over ϕ, θ

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L \cos\theta \sin\theta \, d\phi \, d\theta$$

L → not dependent on direction (isotropic)

$$E = L \iint \ldots$$

$$E = L \int_{\theta=0}^{\pi/2} \Big[\cos\theta \sin\theta \, \phi \Big]_0^{2\pi} d\theta$$

$$\cos\theta \sin\theta \cdot 2\pi$$

$$E = 2\pi L \int_{\theta=0}^{\pi/2} \underbrace{\cos\theta \sin\theta} \, d\theta$$

$\sin\theta \cos\theta = \frac{1}{2}\sin 2\theta$

$$E = 2\pi L \int_{\theta=0}^{\pi/2} \frac{1}{2}\sin 2\theta \, d\theta$$

$$2\pi L \cdot \left[-\frac{1}{4}\cos 2\theta \right]_0^{\pi/2}$$

$\frac{1}{4} - {}^{-}\frac{1}{4}$

$2\pi L \cdot \frac{1}{2}$

E
irradiance
M
exitance
emittance

$\boxed{E = \pi L}$ ↓↓↓↓

$\boxed{M = \pi L}$ ↑↑↑

IRIS : Infra Red Imaging Subsystem

AUSTRALIS-1

700–1100 nm

energy output of sun

consider sun : black body radiator

$T = 5800^\circ K$

ε : 0.99 emissivity

R_s : radius 6.96×10^8 m

D : e→s 1.496×10^{11} m

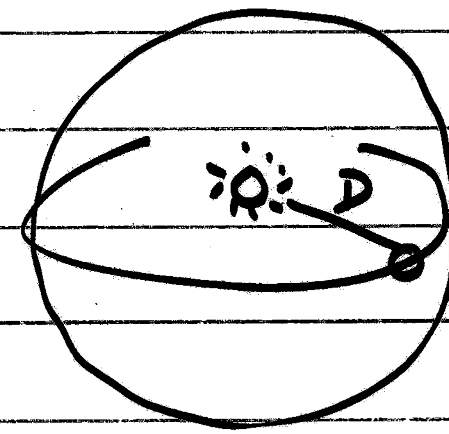
σ : stefan boltzmann constant

5.67×10^{-8} $W m^{-2} K^{-4}$

$$\Phi_{sun} = 4\pi R_s^2 T^4 \sigma \varepsilon$$

$$= 3.87 \times 10^{26} \text{ W} \quad (\text{J/s})$$

all wavelengths
all directions



$$E_e = \frac{\Phi_{sun}}{A} = \frac{\Phi_{sun}}{4\pi D^2}$$

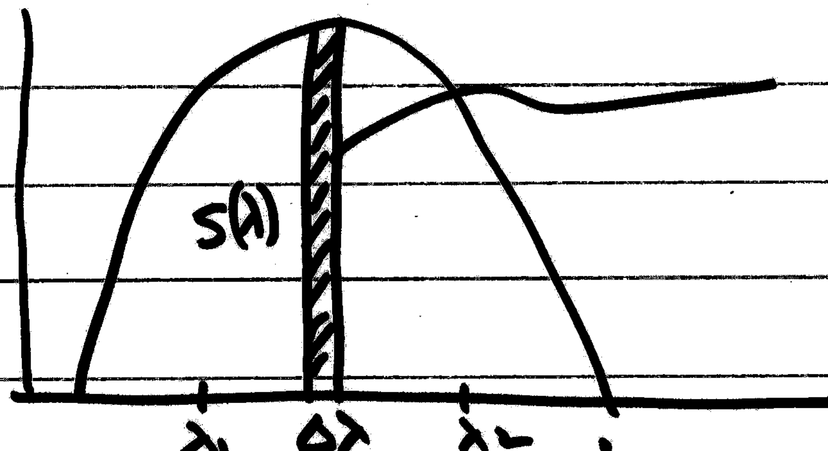
$$E_e = 1.376 \times 10^3 \text{ W/m}^2$$

irradiance @ top of atmosphere

1.376×10^3 W/m^2 all wavelengths

IRIS : 700–1100 nm

S(λ)



λ

$\Delta L = S(\lambda)\,\Delta\lambda$

Spectral radiance

$$L_{\lambda_1\lambda_2} = \sum_{\lambda_i=\lambda_1}^{\lambda_2} S(\lambda_i)\cdot\Delta\lambda$$

$$\boxed{L_{\lambda_1\lambda_1} = \int_{\lambda_1}^{\lambda_2} S(\lambda)\,d\lambda}$$

$$S(\lambda) = \frac{2hc^2}{\lambda^5 \left(e^{hc/\lambda KT} - 1\right)}$$

planck's formula

h : ~~6.26~~ 6.626×10^{-34} Js planck

K : 1.3807×10^{-23} JK^{-1} boltzmann

c : 3×10^8 m/s

T : temp °K

λ : wavelength (m)

I provide Matlab code to numerically integrate

$M = \pi L \cdot \epsilon$

↑ integration

$\phi = 4\pi R_s^2 M$

$E_e = \phi / 4\pi D^2$

now be < 1376 w/m²

$E_e = 377.4 \text{ w/m}^2$

reflectivity or **albedo**
(visible)

forest	~	10%
crops	~	5-15%
grass	~	5-30%
soil	~	5-30%
sand	~	20-40%
ice	~	30-40%
old snow	~	50-70%
new snow	~	75-90%

design maximum
$\underline{\underline{0.7}} = r_d$

$E_e = 377.4\ W/m^2$ restricted to NIR

0.7 albedo

$M_e = E_e \cdot r_d = 264.2\ W/m^2$