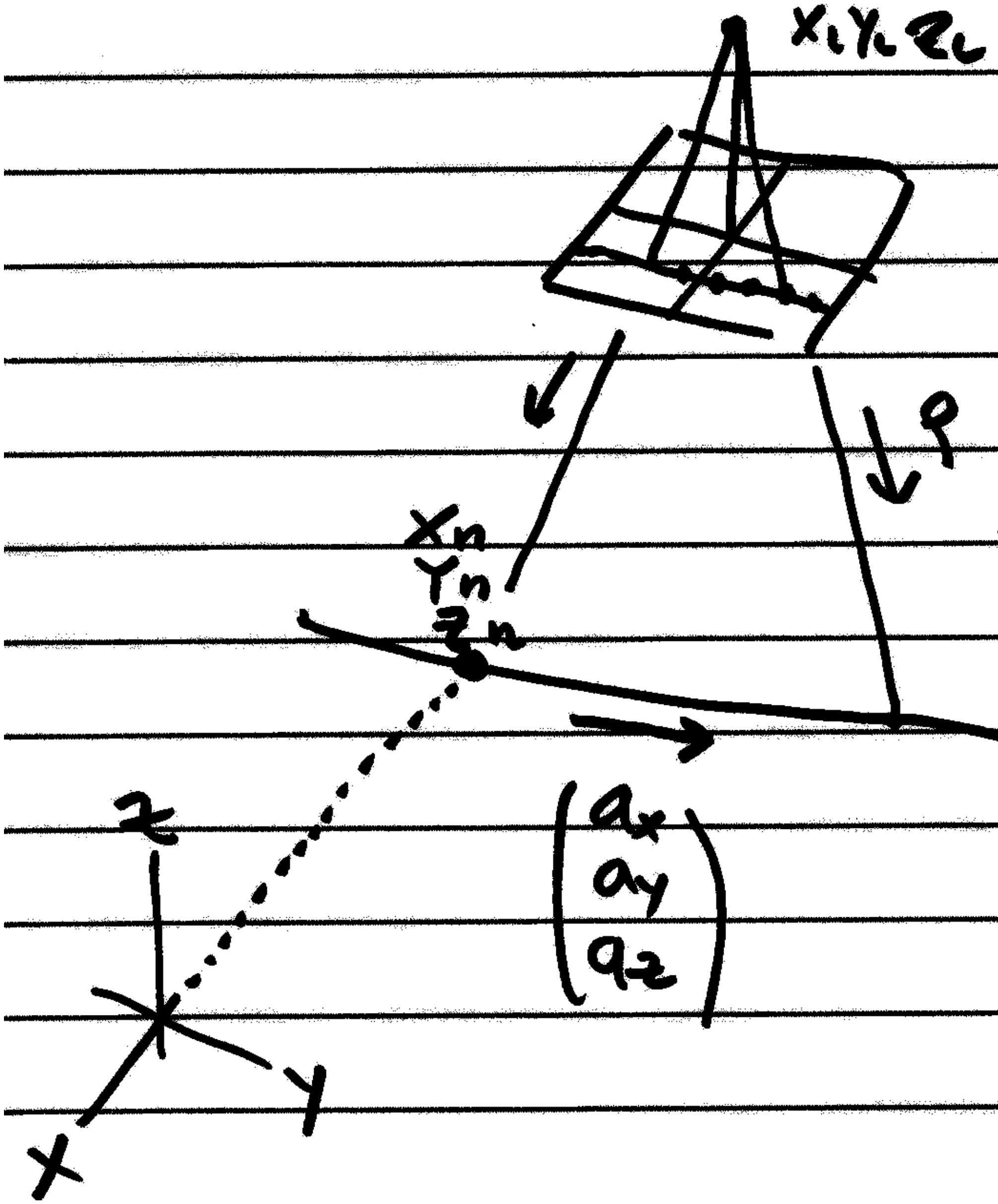
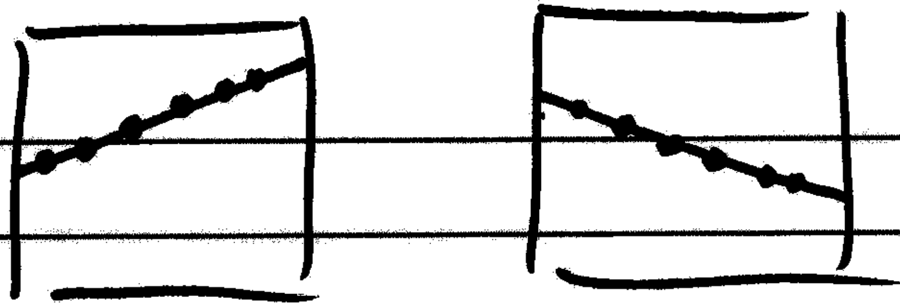
Linear feature : straight L.F.



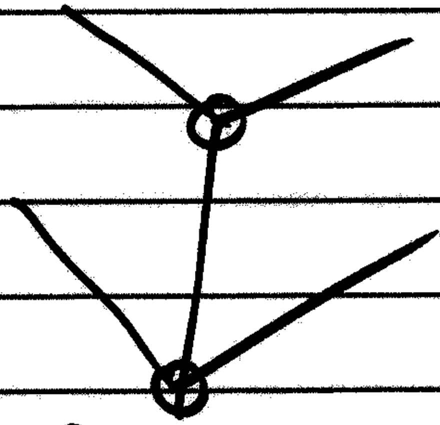
$$\begin{pmatrix} \rho_x \\ \rho_y \\ \rho_z \end{pmatrix} = M^T \begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix}$$

3 vectors are coplanar

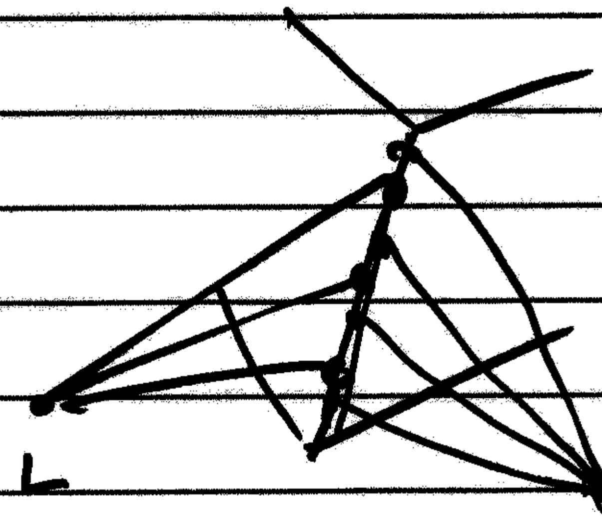
$$\begin{vmatrix} \rho_x & a_x & X_n - X_L \\ \rho_y & a_y & Y_n - Y_L \\ \rho_z & a_z & Z_n - Z_L \end{vmatrix} = 0$$



stereo digitize
straight linear feature



If you are
lucky, 2
vertices define
line, otherwise
you need linear feature
equation



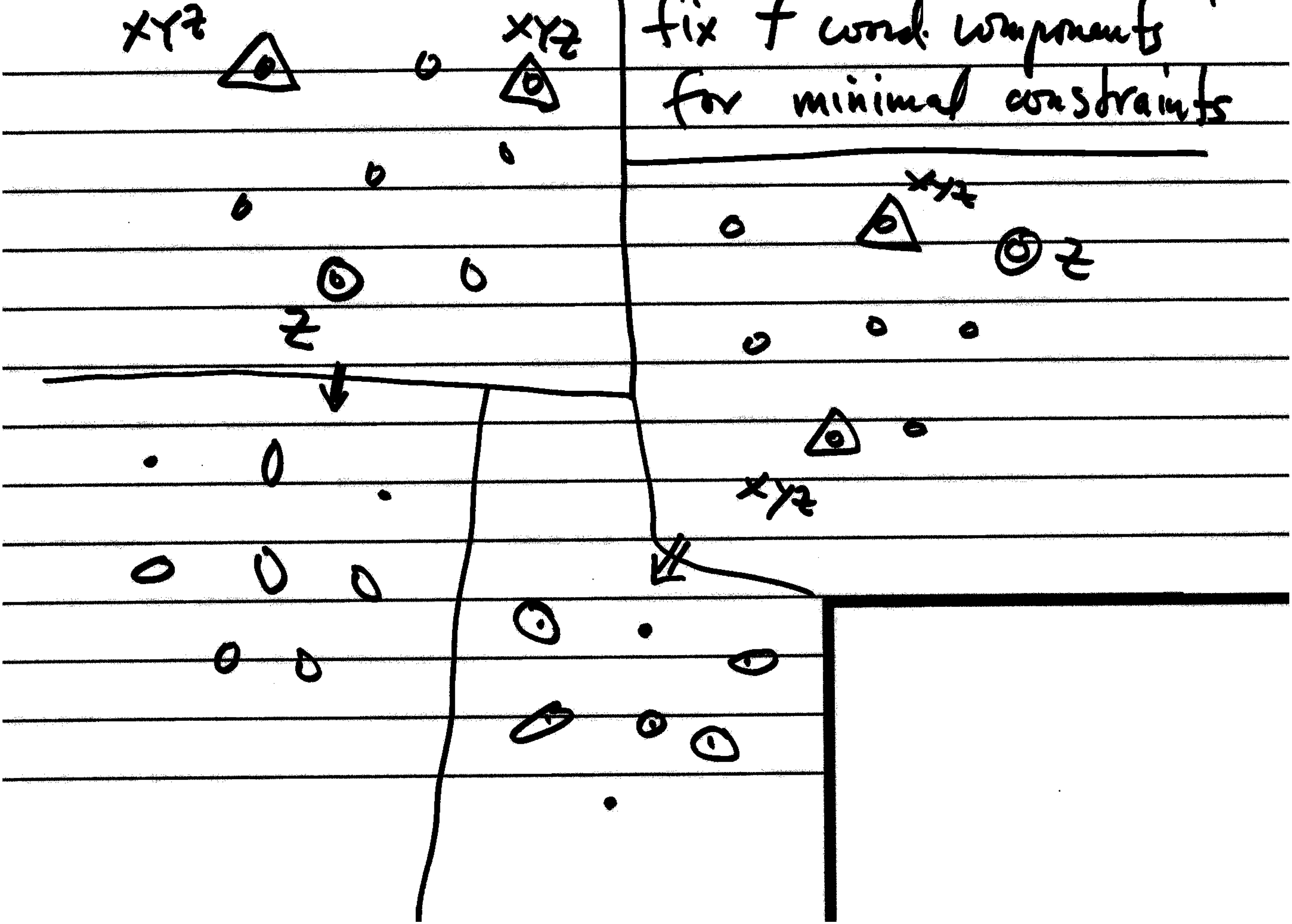
not necessarily conjugate points

minimal constraints

tie network of object points to reference coordinate system without influencing internal geometry of network.

1. verify quality of measurements + conformance to model
2. verify link to ref. coordinate system.

fix 7 coord. components
for minimal constraints



avoid arbitrary assignment of GCP —
distribute constraints among all points

free net adjustment
inner constraint solution
⋮

$X_1 = xxx \quad Y_1 = yyy \quad Z_1 = zzz$
$X_2 = xxx \quad Y_2 = yyy \quad Z_2 = zzz$
.
$Z_3 = zzz$

$C\Delta = g$, N: norm. equ.

$$\begin{bmatrix} N & C^T \\ \underset{=}{C} & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ \lambda \end{bmatrix} = \begin{bmatrix} t \\ g \end{bmatrix}$$

inner constraint solution

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 1 & \\ 0 & Z_1 & -Y_1 & 0 & Z_2 & -Y_2 & \\ -Z_1 & 0 & X_1 & -Z_2 & 0 & X_2 & \\ Y_1 & -X_1 & 0 & Y_2 & -X_2 & 0 & \\ X_1 & Y_1 & Z_1 & X_2 & Y_2 & Z_2 & \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \\ \Delta Z_1 \\ \Delta X_2 \\ \Delta Y_2 \\ \Delta Z_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underbrace{\qquad}_{1}$ $\underbrace{\qquad}_{2}$

No net shift

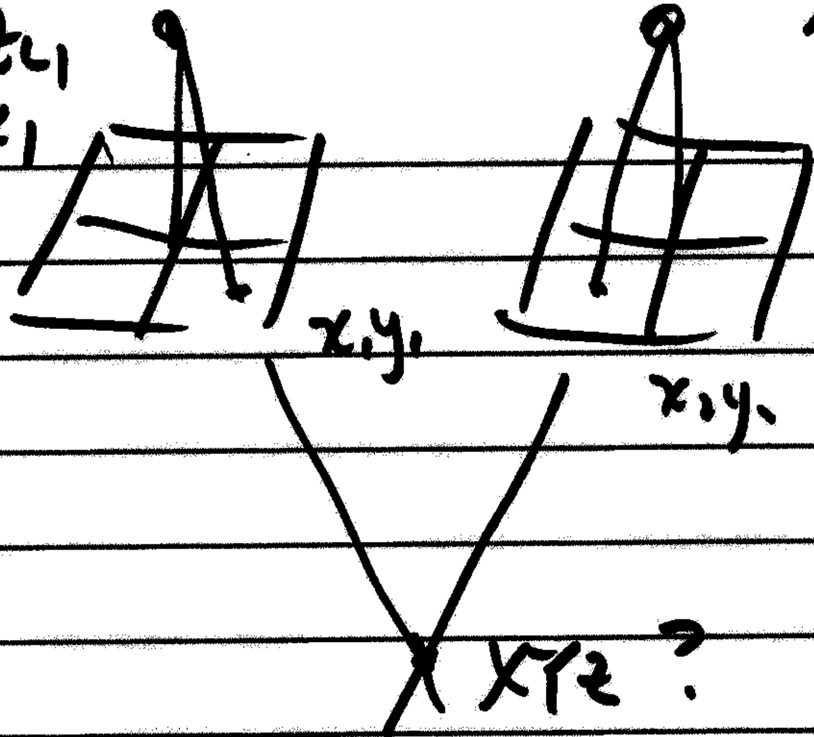
No net rotation

No net scale change

$X_{c1} Y_{c1} Z_{c1}$
$\omega_1 \varphi_1 \kappa_1$

$X_{c2} Y_{c2} Z_{c2}$
$\omega_2 \varphi_2 \kappa_2$

f, x_0, y_0



$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}$$

$$\frac{1}{\lambda} \boxed{M^T \begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix}} = \begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}$$

$$\frac{u}{w} = \frac{X-X_c}{Z-Z_c}$$

$$\frac{v}{w} = \frac{Y-Y_c}{Z-Z_c}$$

Review Linear intersection model

$X = X_L + (Z - Z_L)\, u/w$, let $c_1 = u/w$

$Y = Y_L + (Z - Z_L)\, v/w$, let $c_2 = v/w$

$X = X_L + c_1 (Z - Z_L) = X_L + c_1 Z - c_1 Z_L$

$Y = Y_L + c_2 (Z - Z_L) = Y + c_2 Z - c_2 Z_L$

$X - c_1 Z = X_L - c_1 Z_L$

$Y - c_2 Z = Y_L - c_2 Z_L$

Linear intersection equations

$$\begin{bmatrix} 1 & 0 & -c_1 \\ 0 & 1 & -c_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{pmatrix} X_L - c_1 Z_L \\ Y_L - c_2 Z_L \end{pmatrix}$$

2 lin. eq. 3 unk.

for 2 rays / & 2 images

$$\begin{bmatrix} 1 & 0 & -c_{1_1} \\ 0 & 1 & -c_{2_1} \\ \hdashline 1 & 0 & -c_{1_2} \\ 0 & 1 & -c_{2_2} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{L_1} - c_{1_1} Z_{L_1} \\ Y_{L_1} - c_{2_1} Z_{L_1} \\ X_{L_2} - c_{1_2} Z_{L_2} \\ Y_{L_2} - c_{2_2} Z_{L_2} \end{bmatrix}$$

$$\underset{4,3}{B} \qquad \Delta \qquad f$$

$$\Delta = (B^T B)^{-1} B^T f$$

check misclosure :

take XYZ of intersected point + project back into images

$$x = x_0 - f\,\frac{m_{11}(X - X_c) + m_{12}(Y - Y_c) + \cdots}{m_{31}(X - X_c) + \cdots}$$

$$y = y_0 - f\,\frac{m_{21}(X - X_c) + m_{22}(Y - Y_c) + \cdots}{m_{31}(X - X_c) + \cdots}$$

do for both images

check for consistency

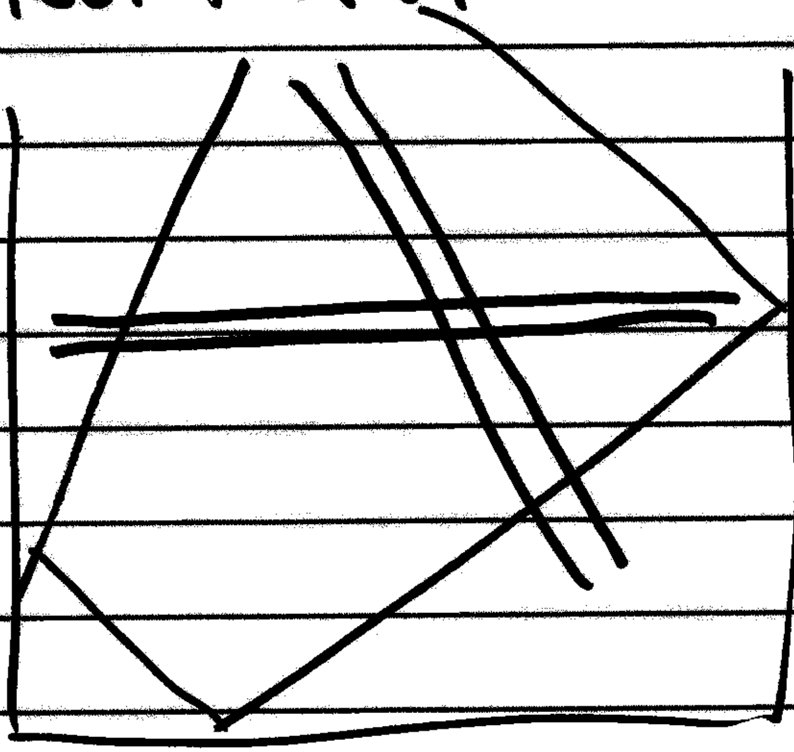
<u>lens distortion</u>

22-11

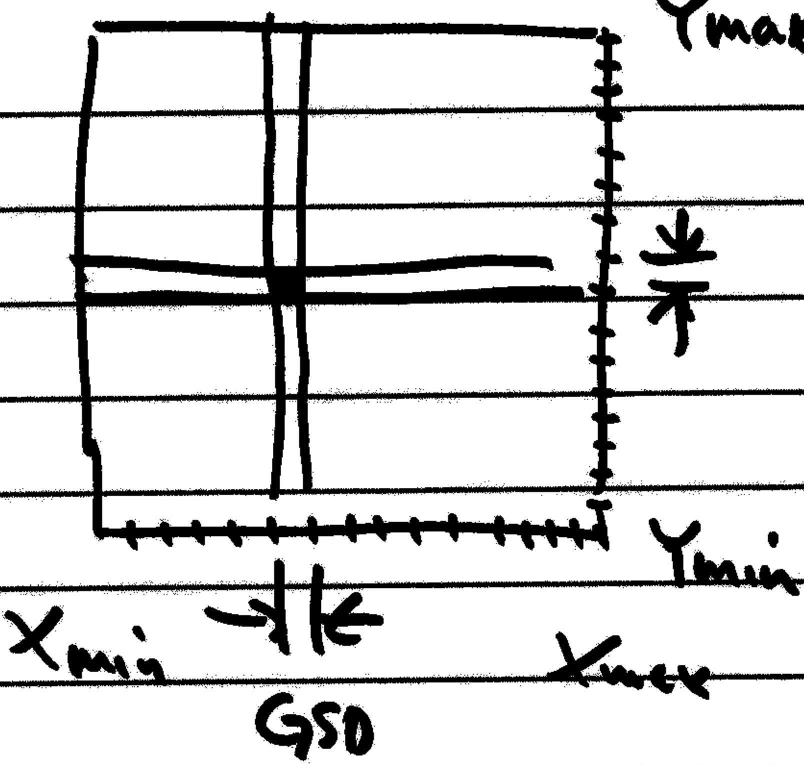
rotim.m : <u>map1.jpg</u>

<u>stereo pair</u> : put into document

rectification : send actual image



22-12



$$\#rows = (Y_{max} - Y_{min}) / GSD$$

$$\#cols = (X_{max} - X_{min}) / GSD$$

$Z = \underline{202}$

$$F_x = x - x_0 + f\frac{u}{w} = 0$$

$$F_y = y - y_0 + f\frac{v}{w} = 0$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M\begin{pmatrix} x - x_c \\ y - y_c \\ z - z_c \end{pmatrix}$$

$$= M\begin{pmatrix} x \\ y \\ z \end{pmatrix} - M\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}$$

$$\frac{d}{dx}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$\frac{\partial}{\partial y}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} m_{12} \\ m_{22} \\ m_{32} \end{pmatrix}$$

$$\frac{\partial}{\partial z}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} m_{13} \\ m_{23} \\ m_{33} \end{pmatrix}$$

Linearization of the non-linear intersection equations

$$\frac{\partial F_x}{\partial p} = f\left(\frac{w\,du - u\,dw}{w^2}\right)$$

$$\frac{\partial F_x}{\partial x},\ \frac{\partial F_x}{\partial y},\ \frac{\partial F_x}{\partial z}$$

$$= \frac{f}{w}\left(\frac{du}{dp} - \frac{u}{w}\frac{dw}{dp}\right)$$

x, y, z

$$\frac{\partial F_y}{\partial p} = f\left(\frac{w\,dv - v\,dw}{w^2}\right)$$

$$\frac{\partial F_y}{\partial x},\ \frac{\partial F_y}{\partial y},\ \frac{\partial F_y}{\partial z}$$

$$= \frac{f}{w}\left(\frac{dv}{dp} - \frac{v}{w}\frac{dw}{dp}\right)$$

x, y, z

Linearized Coord. Eqns :

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{bmatrix} \partial F_x/\partial x & \partial F_x/\partial y & \partial F_x/\partial z \\ \partial F_y/\partial x & \partial F_y/\partial y & \partial F_y/\partial z \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \begin{pmatrix} -F_x \\ -F_y \end{pmatrix}$$

Linearized form of the nonlinear intersection equations

$$\underset{4\times 3}{\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}} \underset{3\times 1}{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}} = \underset{4\times 1}{\underset{f}{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}}}$$