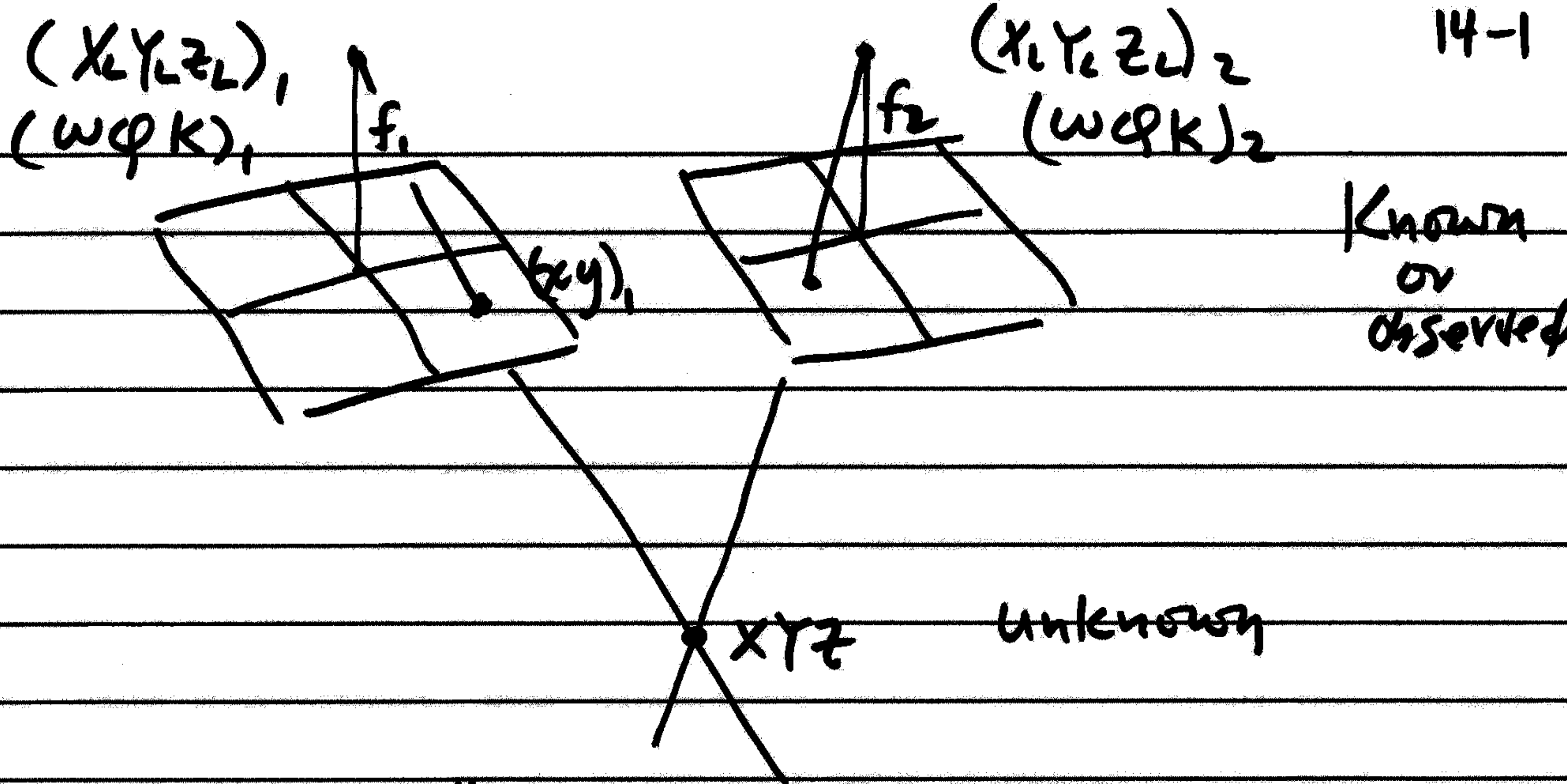


$$F_x = x - x_0 + f\frac{u}{w} = 0$$

$$F_y = y - y_0 + f\frac{v}{w} = 0$$

$$M\begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Know :

$$\Big[\underbrace{X_L Y_L Z_L\, \omega \varphi \kappa}_{\text{Exterior}}\ \underbrace{x_o y_o f}_{\text{Interior}}\Big]_{1+2}$$

unknowns : XYZ

observations : xy_1 , xy_2

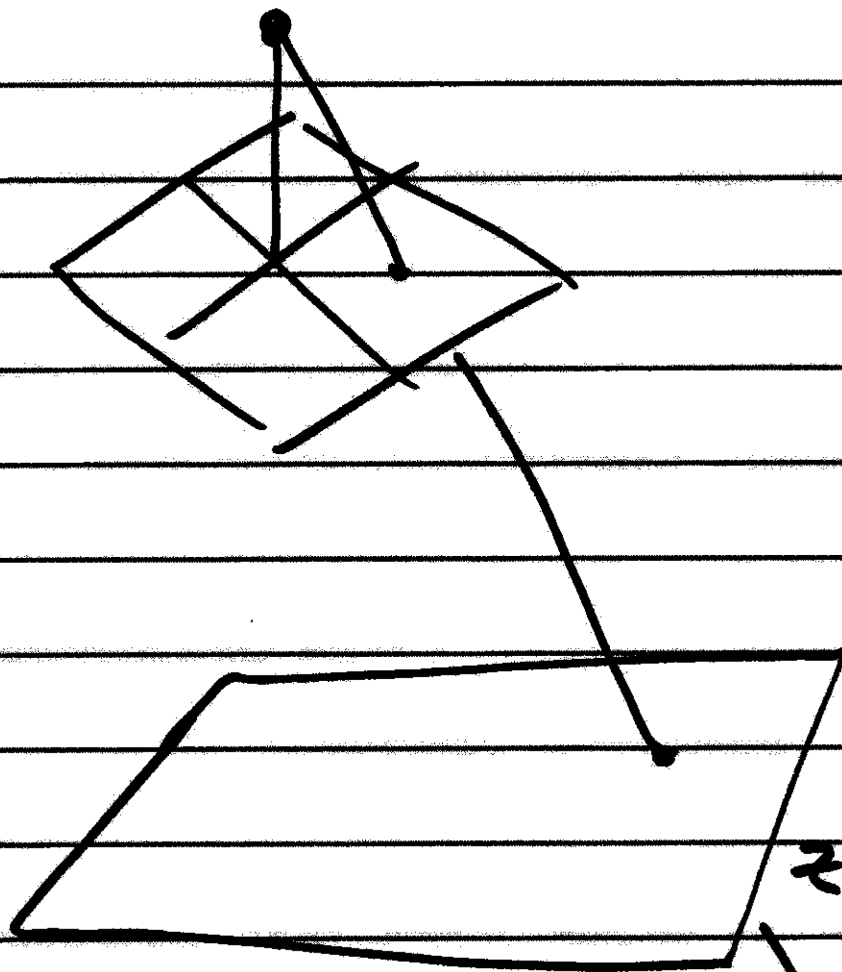
$$\begin{bmatrix} V_{x_1} \\ V_{y_1} \\ V_{x_2} \\ V_{y_2} \end{bmatrix} + \begin{bmatrix} \partial F_{x_1}/\partial x & \partial F_{x_1}/\partial y & \partial F_{x_1}/\partial z \\ \partial F_{y_1}/\partial x & \partial F_{y_1}/\partial y & \partial F_{y_1}/\partial z \\ \partial F_{x_2}/\partial x & \partial F_{x_2}/\partial y & \partial F_{x_2}/\partial z \\ \partial F_{y_2}/\partial x & \partial F_{y_2}/\partial y & \partial F_{y_2}/\partial z \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} -F_{x_1} \\ -F_{y_1} \\ -F_{x_2} \\ -F_{y_2} \end{bmatrix}$$

$$V + B \Delta = f$$

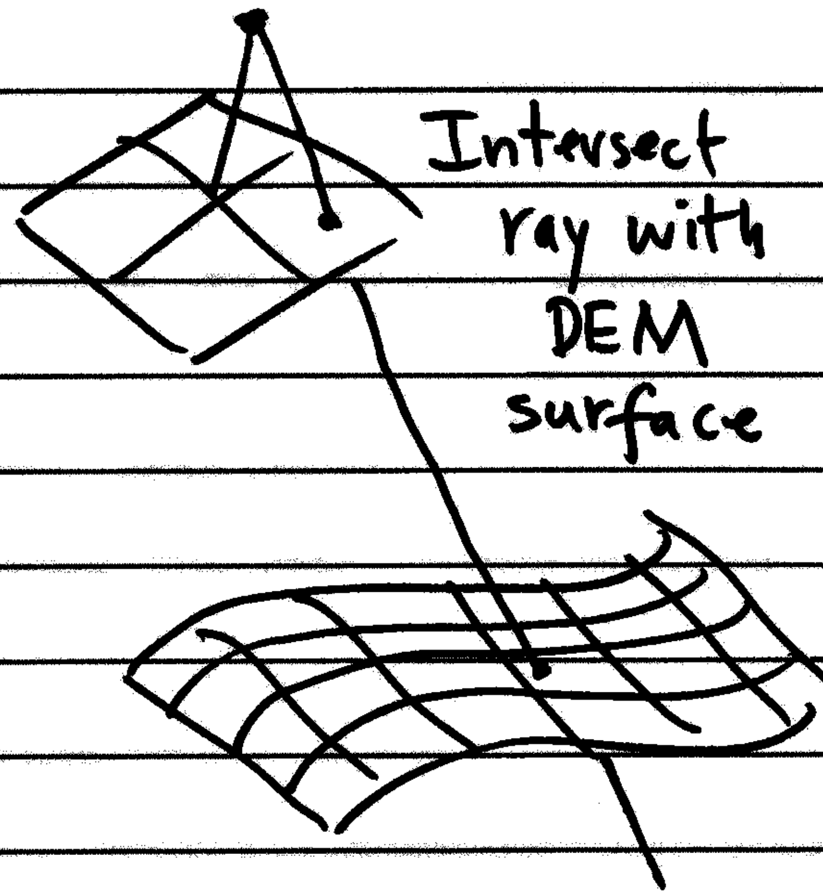
need initial approximation

$x^0_1 \; y^0_1 \; z^0$

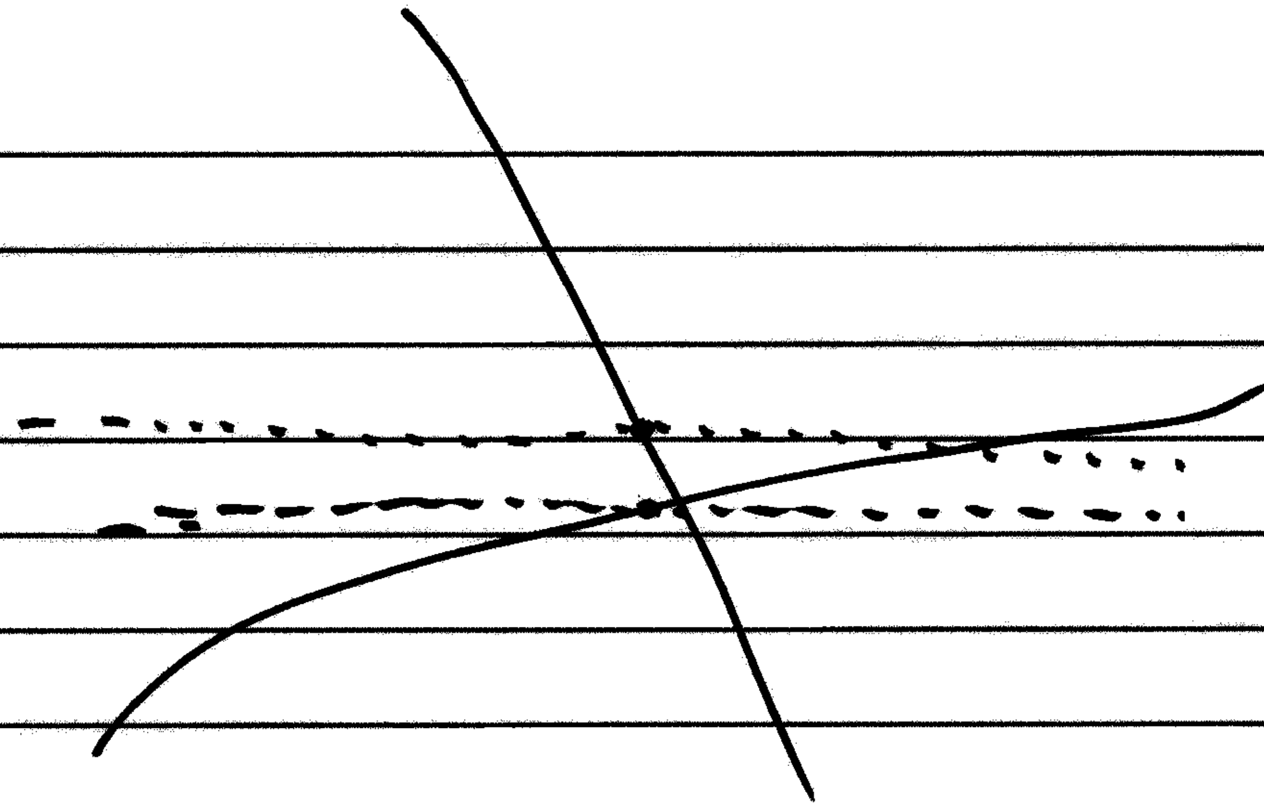
iterate until converged.



Intersect ray with horizontal plane



Intersect ray with DEM surface



Iterative application of ray-plane intersection
to solve the ray-DEM
intersection problem

$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} x-x_c \\ y-y_c \\ z-z_c \end{pmatrix}$$

$$\frac{1}{\lambda} M^T \begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \begin{pmatrix} x-x_c \\ y-y_c \\ z-z_c \end{pmatrix}$$

$$\frac{1}{\lambda} \begin{bmatrix} m_{11}(x-x_0)+m_{21}(y-y_0)+m_{31}(-f) \\ m_{12}(x-x_0)+m_{22}(y-y_0)+m_{32}(-f) \\ m_{13}(x-x_0)+m_{23}(y-y_0)+m_{33}(-f) \end{bmatrix} = \begin{pmatrix} x-x_c \\ y-y_c \\ z_A-z_c \end{pmatrix}$$

$$\frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} x-x_c \\ y-y_c \\ z_A-z_c \end{pmatrix}$$

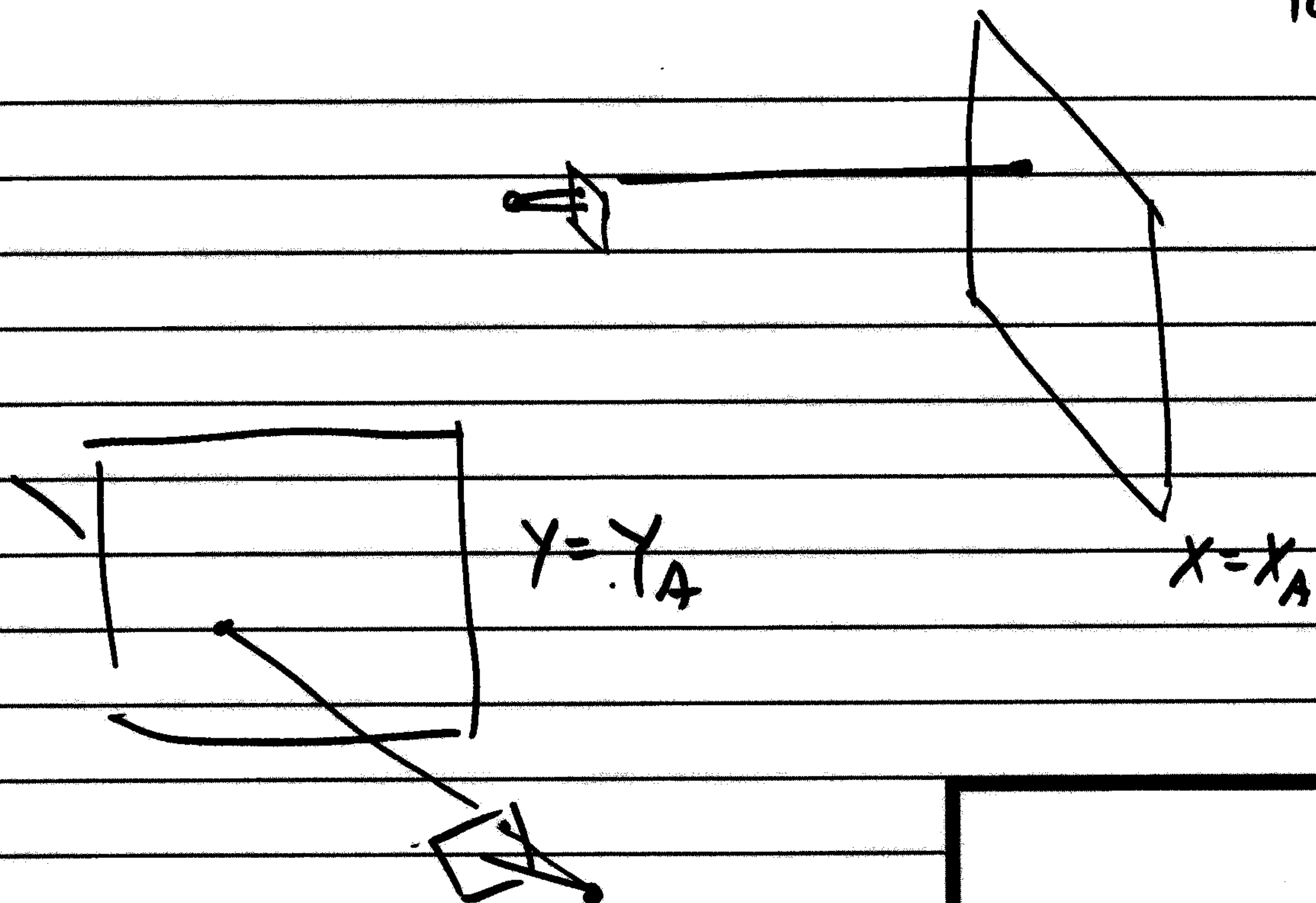
$$\frac{u}{w} = \frac{x - x_L}{z_A - z_L}, \quad x - x_L = (z_A - z_L)\frac{u}{w}$$

$$\frac{v}{w} = \frac{y - y_L}{z_A - z_L}, \quad y - y_L = (z_A - z_L)\frac{v}{w}$$

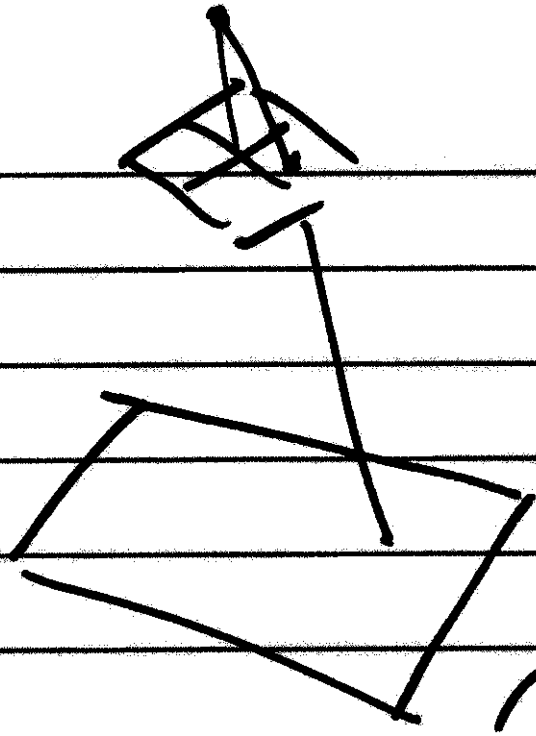
$$x = x_L + (z_A - z_L)\frac{u}{w}$$

$$y = \underbrace{y_L + (z_A - z_L)\frac{v}{w}}$$

Equations to intersect ray with $z = z_A$ plane



By similar approach you can intersect ray with fixed planes $X = X_A$, or $Y = Y_A$.



$$X = X_L + (Z - Z_L)\frac{u}{w}$$

$$Y = Y_L + (Z - Z_L)\frac{v}{w}$$

$$\underline{Z = a_0 + a_1 X + a_2 Y}$$

Equation of tilted plane

$$X = X_L + (a_0 + a_1 X + a_2 Y - Z_L)\frac{u}{w}$$

$$Y = Y_L + (a_0 + a_1 X + a_2 Y - Z_L)\frac{v}{w}$$

Intersect ray with tilted plane

$$X = X_L + (Z - Z_L)\left(\frac{u}{w}\right) \quad c_1$$

$$Y = Y_L + (Z - Z_L)\left(\frac{v}{w}\right) \quad c_2$$

$$X = \underline{X_L} + c_1 Z - \underline{c_1 Z_L}$$

$$Y = \underline{Y_L} + c_2 Z - \underline{c_2 Z_L}$$

unk: $X\ Y\ Z$

$$X - c_1 Z = X_L - c_1 Z_L \quad \cancel{\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}}$$

$$Y - c_2 Z = Y_L - c_2 Z_L$$

Develop a linear version of the intersection equation. This will yield 2 linear equations for 1 image.

$$\begin{pmatrix} 1 & 0 & -c_1 \\ 0 & 1 & -c_2 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_L - c_1 Z_L \\ Y_L - c_2 Z_L \end{pmatrix}$$

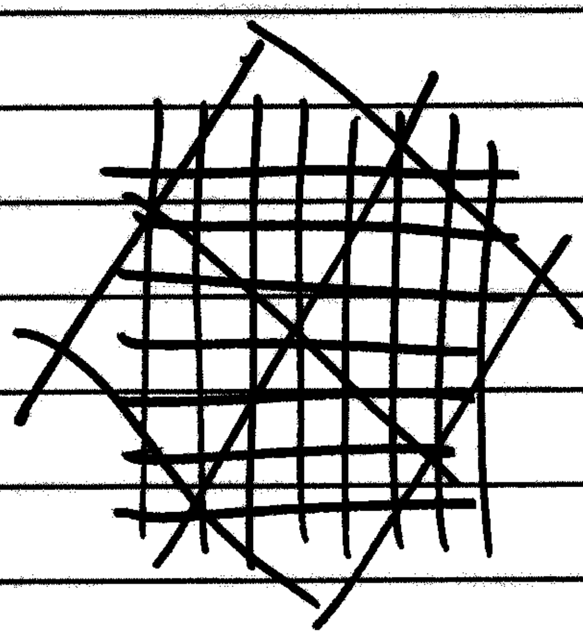
2 linear equations in 3 unknowns

supplement this with 1 or 2 ~~eq~~ linear equations from a second image and you can have linear solution of the intersection problem (you sacrifice correct statistical modeling of the observations.)

Color :

R,G,B   3 chip color
Bayer
Foveon

I,H,S   Intensity, Hue, Saturation

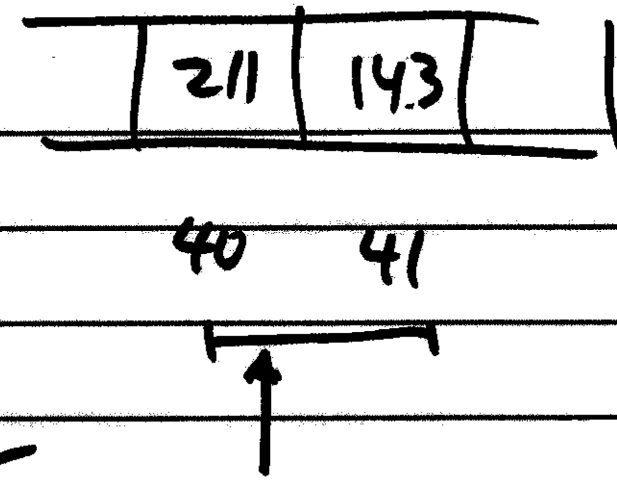


YIQ
YUV

Luminance + Chrominance

NTSC video
JPEG

1D nearest neighbor



8-bit = 0 – 255

10-bit = 0 – 1023

interpolate $x_0 = 40.25$

$40 = \text{round}(40.25)$

$I_{NN}(40.25) = 211$

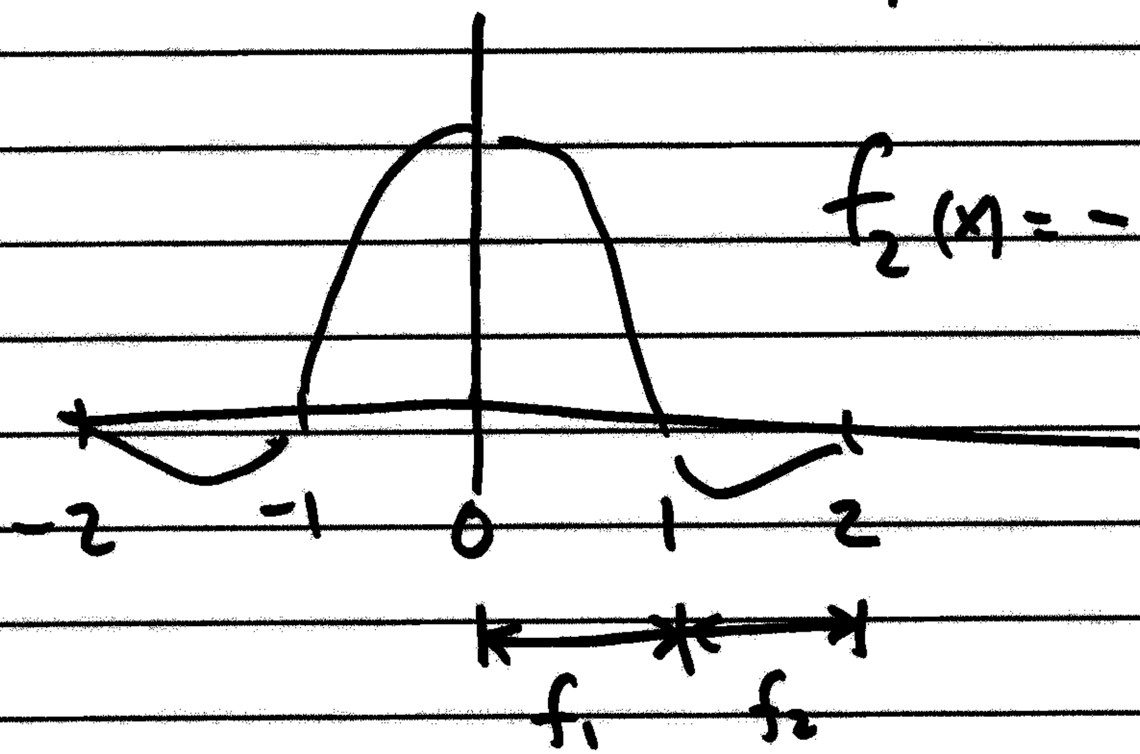
note: see tutorial document on web with these interpolation examples

linear interpolation

$0.25 \times 143 + 0.75 \cdot 211$

$I_L = 194$

Cubic



$f_1(x) = |x|^3 - 2|x|^2 + 1$

$0 < |x| < 1$

$f_2(x) = -|x|^3 + 5|x|^2 - 8|x| + 4$

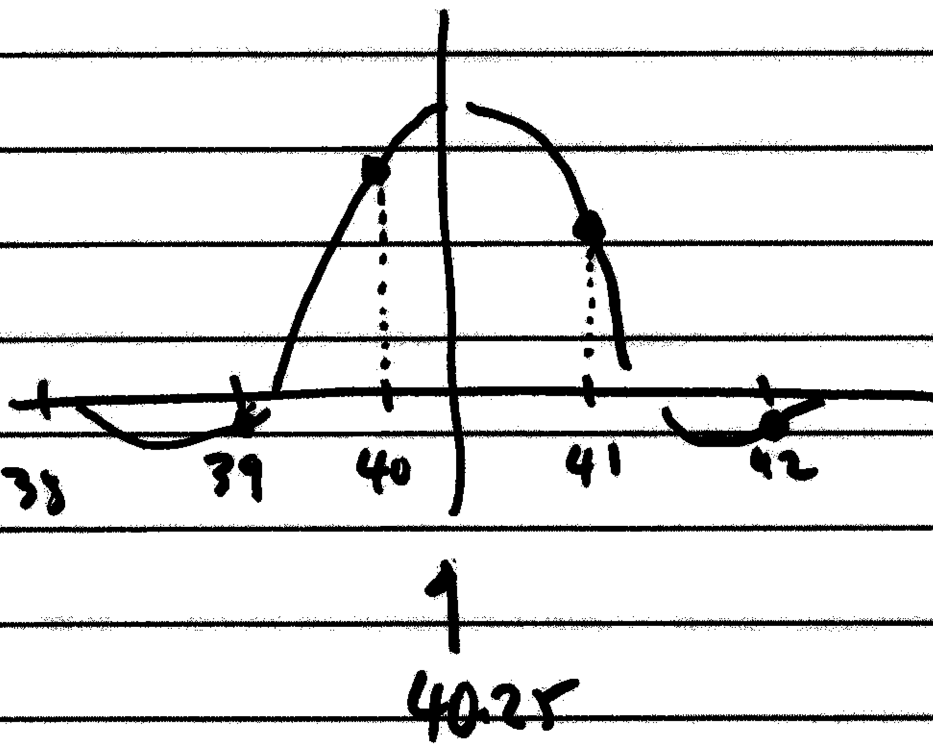
$1 < |x| < 2$

approx Sync

$\frac{\sin x}{x}$

interpolate @ 40.25

237	211	143	138
39	40	41	42



$I_{cc} =$

$I(39) \cdot f(39 - 40.25) +$

$I(40) \cdot f(40 - 40.25) +$

$I(41) \cdot f(41 - 40.25) +$

$I(42) \cdot f(42 - 40.25)$

$I_{cc}(40.25) = 190.3$